

GCSE MATHS

Volume and surface area

Worked Solutions

Q1

★

$$V = 8(5)(3) = 120 \text{ cm}^3$$

Multiply its three perpendicular dimensions.

Q2

★★

$$35h = 210$$

Multiply base area by height.

$$h = 6 \text{ cm}$$

Divide by 35.

Q3

★★

$$A = \frac{1}{2}(6)(4) = 12$$

Find the triangular cross-section area.

$$V = 12(10) = 120 \text{ cm}^3$$

Multiply by prism length.

Q4

★★

$$V = \pi(3)^2(8) = 72\pi \text{ cm}^3$$

Multiply circular area by height.

Q5

★★

$$V = \pi(0.5)^2(1.2) = 0.3\pi \text{ m}^3$$

Calculate volume in cubic metres.

$$1000V = 300\pi = 942.477\dots$$

Convert cubic metres to litres.

$$\text{capacity} \approx 942 \text{ litres}$$

Round to the nearest litre.

Q6

★★★

$$8\pi r^2 = 98\pi$$

Substitute volume and height.

$$r^2 = 12.25 \Rightarrow r = 3.5 \text{ cm}$$

Divide by 8π and take the positive square root.

Q7

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$$S = 2(6 \times 4 + 6 \times 3 + 4 \times 3) = 108 \text{ cm}^2$$

Each of the three face types occurs twice.

Q8

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$$S = 10(6) + 2(10)(4) + 2(6)(4)$$

Include the base and four walls.

$$S = 188 \text{ cm}^2$$

Add the five face areas.

Q9

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$$S = 2\pi(2)^2 + 2\pi(2)(5)$$

Include both circular ends and the curved surface.

$$S = 8\pi + 20\pi = 28\pi \text{ cm}^2$$

Add the areas.

Q10

$$S = \pi(3)^2 + 2\pi(3)(7)$$

Include one circle and the curved wall.

$$S = 51\pi \text{ cm}^2$$

Add.

Q11

$$2A = 2\left(\frac{1}{2}\right)(3)(4) = 12$$

Calculate the two triangular ends.

$$\text{rectangles} = 8(3 + 4 + 5) = 96$$

Multiply prism length by the triangle perimeter.

$$S = 12 + 96 = 108 \text{ cm}^2$$

Add all five faces.

Q12

$$2\pi(4)^2 + 2\pi(4)h = 88\pi$$

Write the surface-area equation.

$$32 + 8h = 88 \Rightarrow h = 7 \text{ cm}$$

Divide by π and solve.

Q13

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$$V = \frac{1}{3}\pi(3)^2(8) = 24\pi \text{ cm}^3$$

Use one third of the matching cylinder volume.

Q14

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$$V = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$$

Cube the radius before multiplying.

Q15

$$\text{curved area} = 2\pi(4)^2 = 32\pi$$

Halve the surface area of a sphere.

$$\text{flat face} = \pi(4)^2 = 16\pi$$

Include the circular base.

$$S = 48\pi \text{ cm}^2$$

Add the exposed areas.

Q16

$$\ell = \sqrt{5^2 + 12^2} = 13$$

Find slant height using Pythagoras.

$$S = \pi(5)(13) + \pi(5)^2 = 90\pi \text{ cm}^2$$

Add curved area and circular base.

Q17

$$V = \frac{1}{3}(6^2)(4) = 48 \text{ cm}^3$$

Multiply base area by perpendicular height and divide by three.

Q18

$$\frac{4}{3}\pi r^3 = 288\pi$$

Substitute the volume.

$$r^3 = 216 \Rightarrow r = 6 \text{ cm}$$

Multiply by 3/4, cancel π and take the cube root.

Q19

(a) $V_{\text{cyl}} = \pi(3)^2(8) = 72\pi$

$$V_{\text{hemi}} = \frac{2}{3}\pi(3)^3 = 18\pi$$

$$V = 90\pi \text{ cm}^3$$

Find cylinder volume.

Use half a sphere.

Add the volumes.

(b) $S = 2\pi(3)(8) + 2\pi(3)^2 + \pi(3)^2$

$$S = 48\pi + 18\pi + 9\pi = 75\pi \text{ cm}^2$$

Count cylinder wall, curved dome and bottom circle.

The circular join is internal.

Q20

$$r_{\text{small}} = 6\left(\frac{1}{2}\right) = 3$$

$$V_{\text{whole}} = \frac{1}{3}\pi(6)^2(12) = 144\pi$$

$$V_{\text{small}} = \frac{1}{3}\pi(3)^2(6) = 18\pi$$

$$V_{\text{remaining}} = 126\pi \text{ cm}^3$$

Use similarity to find the cut radius.

Find the original volume.

Find the removed cone volume.

Subtract.

Q21

(a) $N = 1 + 3 + 2 + 2 + 1 + 4 = 13$

$$V = 13(1^3) = 13 \text{ cm}^3$$

Add the column heights.

Each cube has unit volume.

(b) taller of 1 and 2 : 2

taller of 3 and 1 : 3

taller of 2 and 4 : 4

$$2, 3, 4 \text{ cm}$$

Take the highest column in each line of sight.

The final elevation column has height 4.

State the front elevation heights from left to right.

(c) top and bottom = $6 + 6 = 12$

$$\text{outer vertical faces} = 6 + 7 + 3 + 6 = 22$$

$$\text{steps} = 3 + 4 + 5 = 12$$

$$S = 12 + 22 + 12 = 46 \text{ cm}^2$$

There is one exposed top and bottom per column.

Sum heights along back, front, left and right edges.

Height differences total 3 along the back row, 4 along the front row and 5 between the rows.

Add all exposed faces.