

GCSE MATHS

Volume and surface area

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Prisms and cylinders	Q1–6	***	3
2. Surface area	Q7–12	***	6
3. Cones, spheres and pyramids	Q13–18	***	10
4. Composite solids and views	Q19–21	****	14
Solving principles			18

Topic recap

Prisms

$V = Ah$, where A is cross-sectional area and h is the perpendicular length. A cylinder is a circular prism:
 $V = \pi r^2 h$.

Surface area

Add the areas of all exposed faces. A cylinder has curved area $2\pi r h$; each circular end has area πr^2 .

Other solids

Cone: $V = \frac{1}{3}\pi r^2 h$, curved area $\pi r \ell$ (slant height ℓ). Sphere: $V = \frac{4}{3}\pi r^3$, area $4\pi r^2$. Pyramid: $V = \frac{1}{3}Ah$.

Views

A plan looks vertically down. An elevation looks horizontally from the stated direction. For aligned stacks of cubes, each elevation column shows the greatest height behind it.

A calculator may be used. Keep π in exact answers; round only when asked.

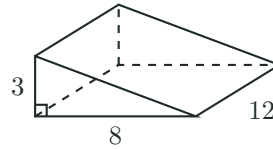
Solid diagrams are not to scale. Calculate from the labelled dimensions.

1. Prisms and cylinders

Example Problem 1

**

A right triangular prism has cross-section base 8 cm and height 3 cm, and length 12 cm. Find its volume.



The idea

1. Use the area of one end, then multiply by the distance to the other end.

Worked Solution

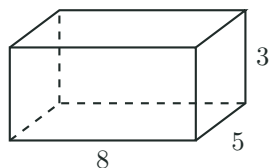
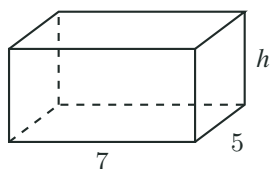
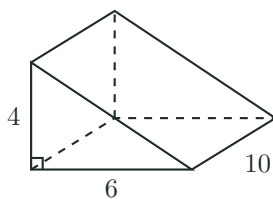
$$A = \frac{1}{2}(8)(3) = 12 \text{ cm}^2$$

Find the cross-sectional area.

$$V = 12(12) = 144 \text{ cm}^3$$

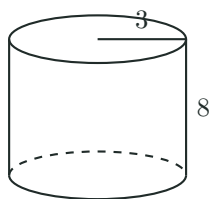
Multiply by the prism length.

1 Prisms and cylinders

Q1 Find the volume of a cuboid measuring 8 cm by 5 cm by 3 cm. ***Q2** A cuboid has volume 210 cm^3 and base 7 cm by 5 cm. Find its height. ****Q3** A triangular prism has a right-angled cross-section with perpendicular sides 6 cm and 4 cm, and length 10 cm. Find its volume. **

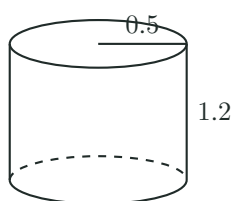
Q4 Find the volume of a cylinder of radius 3 cm and height 8 cm. Give an exact answer.

**

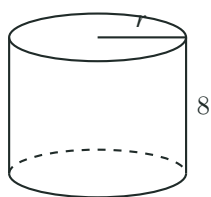


Q5 A cylindrical tank has internal radius 0.5 m and depth 1.2 m. Find its capacity in litres, to the nearest litre.

**



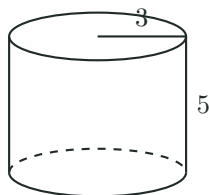
Q6 A cylinder has volume $98\pi \text{ cm}^3$ and height 8 cm. Find its radius.



2. Surface area

Example Problem 2

A closed cylinder has radius 3 cm and height 5 cm. Find its total surface area.



The idea

1. Identify every exposed face before calculating.

Worked Solution

$$\text{ends} = 2\pi(3)^2 = 18\pi$$

$$\text{wall} = 2\pi(3)(5) = 30\pi$$

$$S = 48\pi \text{ cm}^2$$

There are two circles.

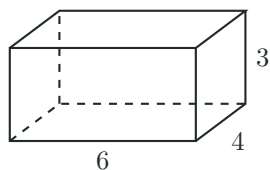
Unroll the curved wall into a rectangle:
circumference by height.

Add the three surfaces.

2 Surface area

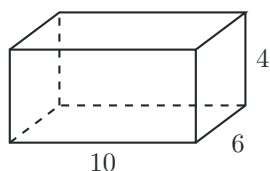
Q7 Find the total surface area of a closed cuboid 6 cm by 4 cm by 3 cm.

**

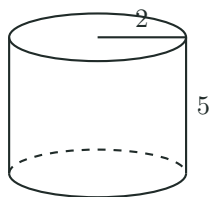


Q8 A box with no lid measures 10 cm by 6 cm by 4 cm internally. Ignore material thickness. Find the area of material needed.

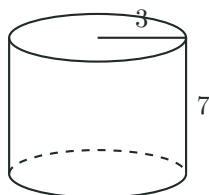
**



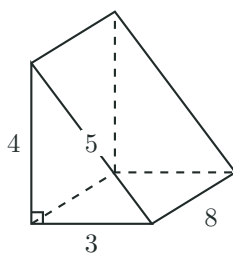
- Q9** Find the total surface area of a closed cylinder with radius 2 cm and height 5 cm. Give an exact answer. **



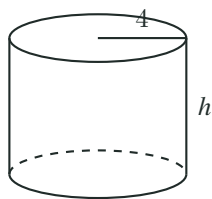
- Q10** A cylindrical cup is open at the top. Its radius is 3 cm and height is 7 cm. Ignore thickness. Find its outside area, including the base. ***



- Q11** A closed right triangular prism has cross-section sides 3 cm, 4 cm and 5 cm, and length 8 cm. Find its surface area.



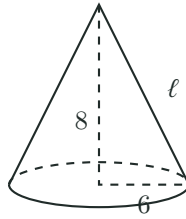
- Q12** A closed cylinder has radius 4 cm and total surface area $88\pi \text{ cm}^2$. Find its height.



3. Cones, spheres and pyramids

Example Problem 3

A cone has radius 6 cm and height 8 cm. Find (a) its volume; (b) its total surface area.



The idea

1. Volume and curved surface area use different heights.

Worked Solution

(a) $V = \frac{1}{3}\pi(6)^2(8) = 96\pi \text{ cm}^3$

Use perpendicular height for volume.

(b) $\ell = \sqrt{6^2 + 8^2} = 10$

Find slant height for the curved surface.

$$S = \pi(6)(10) + \pi(6)^2 = 96\pi \text{ cm}^2$$

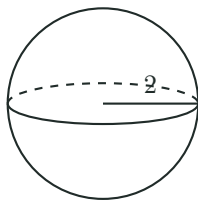
Add the base.

3. Cones, spheres and pyramids

Example Problem 4

**

Find the volume of a sphere of radius 2 cm. Give an exact answer.



The idea

1. A sphere uses the radius cubed. Keep π in the exact answer.

Worked Solution

$$V = \frac{4}{3}\pi r^3$$

Use the sphere formula.

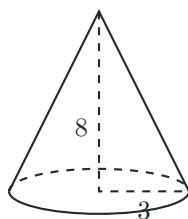
$$V = \frac{4}{3}\pi(2)^3 = \frac{32\pi}{3} \text{ cm}^3$$

Substitute the radius and cube it.

3 Cones, spheres and pyramids

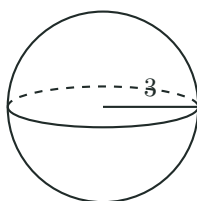
- Q13** Find the volume of a cone of radius 3 cm and perpendicular height 8 cm. Give an exact answer.

**



- Q14** Find the volume of a sphere of radius 3 cm. Give an exact answer.

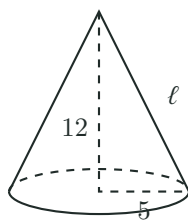
**



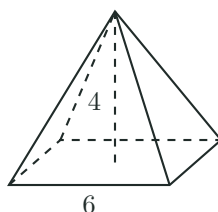
- Q15** A solid hemisphere has radius 4 cm. Find its total surface area, including its flat face.



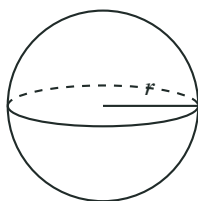
Q16 A cone has radius 5 cm and perpendicular height 12 cm. Find its total surface area.



Q17 A square-based pyramid has base side 6 cm and perpendicular height 4 cm. Find its volume.



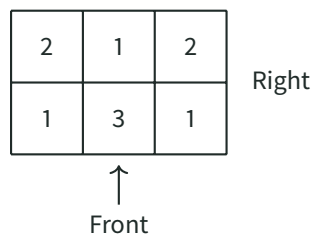
Q18 A sphere has volume $288\pi \text{ cm}^3$. Find its radius.



4. Composite solids and views

Example Problem 5

The plan shows heights of columns of 1 cm cubes. Find the front elevation heights, from left to right.



The idea

1. Look from the front arrow and use the tallest stack in each line of sight.

Worked Solution

taller of 2 and 1 : 2

taller of 1 and 3 : 3

taller of 2 and 1 : 2

2, 3, 2 cm

Compare front and back heights in each column.

The third column has maximum height 2.

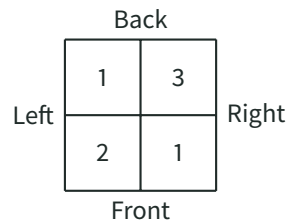
State the three elevation heights.

4. Composite solids and views

Example Problem 6

★★★★

The plan shows column heights in a solid made from 1 cm cubes, with no gaps underneath. Find its total surface area, including the bottom.



The idea

1. Count one top and one bottom face for each column.
2. Add the column heights along each outside edge.
3. Between neighbouring columns, only the difference in height is exposed.

Worked Solution

$$A_{\text{top and bottom}} = 2 \times 4 = 8$$

There are four columns, each with unit top and bottom.

$$A_{\text{outside}} = (1 + 3) + (2 + 1) + (1 + 2) + (3 + 1) = 14$$

Add the back, front, left and right edge heights.

$$A_{\text{steps}} = 2 + 1 + 1 + 2 = 6$$

Count each neighbouring pair once.

$$A = 8 + 14 + 6 = 28 \text{ cm}^2$$

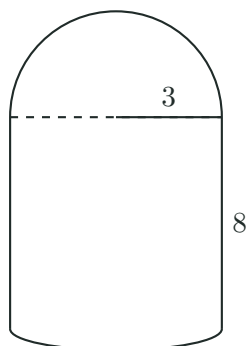
Add all exposed faces.

4 Composite solids and views

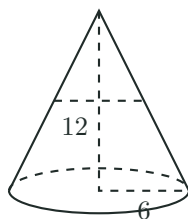
Q19 A cylinder of radius 3 cm and height 8 cm is topped by a hemisphere of radius 3 cm.

(a) Find the volume of the solid.

(b) Find its outside surface area, including the bottom.



- Q20** A cone of radius 6 cm and height 12 cm has its top removed by a cut parallel to its base, halfway up its height. Find the remaining volume.



- Q21** The plan shows heights of columns of 1 cm cubes. There are no gaps below a cube.

- (a) Find the total volume.
 (b) State the front elevation heights from left to right.
 (c) Find the surface area of the solid, including its base.

1	3	2
2	1	4

↑
Front

Right

Solving principles

1. Identify the solid

Choose a volume or surface-area formula to match the shape and the requested measure.

2. Count exposed surfaces

An open end has no face. A join inside a composite solid does not contribute to its outside area.

3. Use perpendicular height

The cone volume uses vertical height; its curved area uses slant height.

4. Keep exact values

Retain fractions and π until the final numerical answer.