

GCSE MATHS

Venn diagrams, two-way tables and frequency trees

Worked Solutions

Q1

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$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{3, 6, 9\}$$

$$A \cap B = \{6\}$$

List each set.

Keep the common member.

Q2

★★

$$n(A \cup B) = 7 + 4 + 7 = 18$$

Include all three regions inside at least one circle.

Q3

★★

$$P = \frac{12}{20} = \frac{3}{5}$$

Count the relevant individuals and divide by the total.

Q4

★★

$$P = \frac{6}{30} = \frac{1}{5}$$

Count the relevant individuals and divide by the total.

Q5

★★

$$A = \{1, 2, 3, 4, 6, 12\}$$

$$A \cap B' = \{1, 2, 4\}$$

List all factors.

Remove the factors that are multiples of 3.

Q6

★★

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cap B) = 0$$

An overlap is counted in both set totals.

No subtraction is needed precisely when there is no overlap.

Q7

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$$n(F \cup S) = 23 + 18 - 9 = 32$$

Subtract the overlap counted twice.

$$n(\text{neither}) = 40 - 32 = 8$$

Subtract the union from the whole group.

Q8

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$$n(T \cup F) = 50 - 7 = 43$$

Find the number playing at least one sport.

$$n(T \cap F) = 28 + 25 - 43 = 10$$

Recover the overlap from the two totals.

Q9

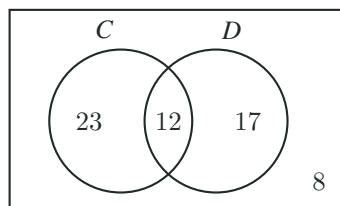
$$n(C) = 35$$

Restrict attention to cat owners.

$$n(C \cap D) = 12$$

$$P = 12/35$$

Use the cat-owner total as denominator.

**Q10**

$$n(A \cap M) = 45 + 38 - (80 - 15) = 18$$

Find the overlap.

$$n(\text{exactly one}) = 45 + 38 - 2(18) = 47$$

Remove the overlap from each subject total.

$$P = 47/80$$

Divide by all pupils.

Q11

$$n(B') = 36 - 18 = 18$$

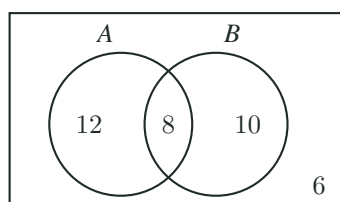
Use the group outside B .

$$n(A \cap B') = 20 - 8 = 12$$

Count the A -only region.

$$P = 12/18 = 2/3$$

Divide within the restricted group.



Q12

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$$n(T \cup C) = 88$$

Exclude the neither group.

$$n(T \cap C) = 62 + 58 - 88 = 32$$

Find the overlap.

$$P = 32/88 = 4/11$$

Use the at-least-one group as the total.

Q13

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$$P(\text{bus}) = 30/70 = 3/7$$

Use the bus-column total and grand total.

Q14

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$$P = \frac{24}{40} = \frac{3}{5}$$

Use only the Year 11 row.

Q15

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$$P = \frac{20}{30} = \frac{2}{3}$$

Use only the bus column.

Q16

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$$\text{Y10 bus} = 40 - 25 = 15$$

Complete Year 10.

$$\text{Y11 bus} = 37 - 15 = 22$$

Use the bus total.

$$\text{Y11 walk} = 50 - 22 = 28$$

Complete Year 11.

$$\text{walk total} = 25 + 28 = 53$$

Add the walking counts.

Q17

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$$\frac{10}{25} = \frac{2}{5}$$

Find the bus proportion in Year 10.

$$\frac{20}{50} = \frac{2}{5}$$

The probability is the same in both year groups,
so the events are independent.

Q18

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$$x/(x+20) = 3/5$$

$$5x = 3x + 60 \Rightarrow x = 30$$

Use the swimmer column.

Solve.

Q19

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$$15 + 15 = 30$$

Add the late counts from both travel groups.

Q20

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$$P = \frac{8}{60} = \frac{2}{15}$$

Use the group specified in the question as the denominator.

Q21

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$$P = \frac{12}{30} = \frac{2}{5}$$

Use the group specified in the question as the denominator.

Q22

$$n(\text{bus}) = 80 - 35 = 45$$

Subtract the walk total.

$$n(\text{bus and late}) = 45 - 36 = 9$$

Subtract on-time bus passengers.

Q23

$$\text{late bus} = 54(1/6) = 9$$

Use the complement of being on time.

$$\text{walkers} = 90 - 54 = 36$$

Find the other travel group.

$$\text{late walk} = 36(1/4) = 9$$

Find late walkers.

$$P = 9/(9 + 9) = 1/2$$

Condition on being late.

Q24

$$\frac{x}{(x+4) + x + 2x + 2} = \frac{1}{5}$$

Add the numbers in all the separate regions to find the total.

$$5x = 4x + 6 \Rightarrow x = 6$$

Cross-multiply and solve.