

## GCSE MATHS

# Venn diagrams, two-way tables and frequency trees

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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# Topic recap

## Set notation

$A \cap B$  means in both sets;  $A \cup B$  means in either or both;  $A'$  means outside  $A$ . The universal set contains all outcomes under consideration.  $n(A)$  is the number of members of  $A$ .

## Venn counts

Enter an overlap before calculating either single-set region. A set total includes its overlap.

## Two-way tables

Every individual belongs to one internal cell. Add across rows and down columns to find totals.

## Conditional probability

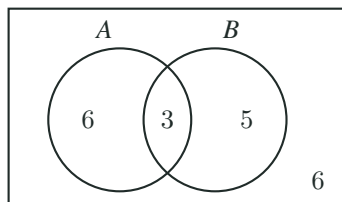
For a conditional probability, restrict the total to the stated subgroup, then count its successful outcomes. A frequency tree contains counts; branches from a node add to that node's count.

# 1. Sets and Venn diagrams

## Example Problem 1

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Use the diagram to find  $P(A \cup B)$  for a random individual.



### The idea

1. The union includes the overlap once.

### Worked Solution

$$n(A \cup B) = 6 + 3 + 5 = 14$$

$$N = 6 + 3 + 5 + 6 = 20$$

$$P(A \cup B) = 14/20 = 7/10$$

Count the three regions inside the circles.

Include the outside region in the population.

Divide.

**1 Sets and Venn diagrams**

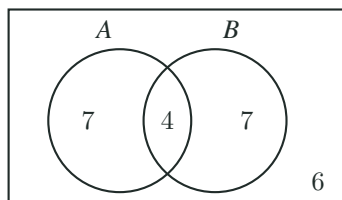
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- Q1** The universal set is  $\{1, 2, \dots, 10\}$ .  $A$  is the set of even numbers and  $B$  is the set of multiples of 3. List  $A \cap B$ .

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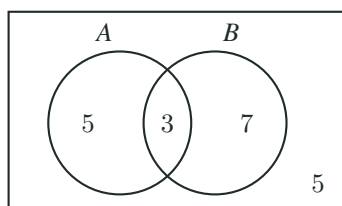
- Q2** Use the Venn diagram to find  $n(A \cup B)$ .

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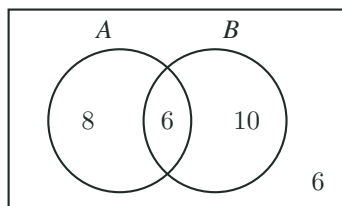
- Q3** Use the Venn diagram to find  $P(A')$  when one individual is chosen at random.

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**Q4** Use the Venn diagram to find  $P(A \cap B)$  for a random individual.

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**Q5** The universal set is  $\{1, 2, \dots, 12\}$ . A contains the factors of 12 and B contains the multiples of 3. List  $A \cap B'$ .

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**Q6** When is  $n(A \cup B) = n(A) + n(B)$  true?

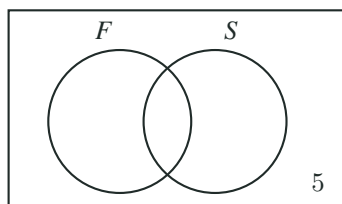
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## 2. Completing Venn diagrams

### Example Problem 2

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Of 30 pupils, 18 study French ( $F$ ), 15 study Spanish ( $S$ ), and 5 study neither. Find how many study French only.



### The idea

1. Fill the overlap before either single-subject region.

### Worked Solution

$$n(F \cup S) = 30 - 5 = 25$$

Find the number inside at least one circle.

$$n(F \cap S) = 18 + 15 - 25 = 8$$

Subtract the union from the sum of set totals.

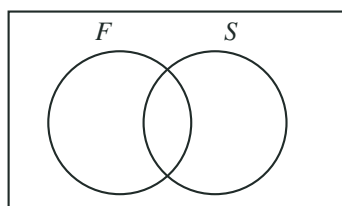
$$n(\text{F only}) = 18 - 8 = 10$$

Remove the overlap from the French total.

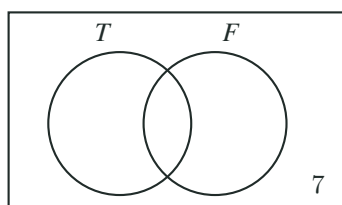
## 2 Completing Venn diagrams

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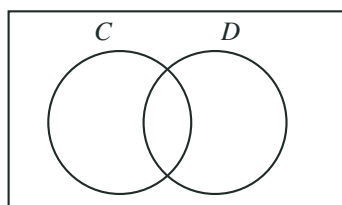
- Q7** Of 40 pupils, 23 study French ( $F$ ), 18 study Spanish ( $S$ ), and 9 study both. Find how many study neither. \*\*



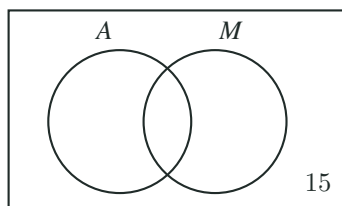
- Q8** Of 50 pupils, 28 play tennis ( $T$ ), 25 play football ( $F$ ), and 7 play neither. Find how many play both. \*\*



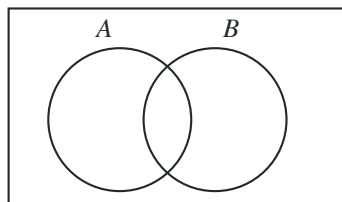
- Q9** Complete the Venn diagram. Of 60 people, 35 own a cat ( $C$ ), 29 own a dog ( $D$ ), and 12 own both. A cat owner is chosen at random. Find the probability they also own a dog. \*\*\*



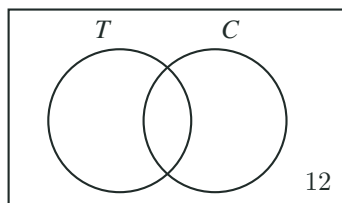
- Q10** Of 80 pupils, 45 take art ( $A$ ), 38 take music ( $M$ ), and 15 take neither. Find the probability that a random pupil takes exactly one of art and music. \*\*\*



- Q11** Complete the Venn diagram. In a group of 36,  $n(A) = 20$ ,  $n(B) = 18$  and  $n(A \cap B) = 8$ . A person outside  $B$  is selected at random. Find the probability that they are in  $A$ . \*\*\*



- Q12** In a group of 100, 62 like tea ( $T$ ), 58 like coffee ( $C$ ), and 12 like neither. Find the probability that a randomly chosen person who likes at least one drink likes both. \*\*\*\*



## 3. Two-way tables

### Example Problem 3

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Using the table, find the probability a randomly selected Year 10 pupil travels by bus.

|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 18   | 12  | 30    |
| Year 11 | 22   | 18  | 40    |
| Total   | 40   | 30  | 70    |

### The idea

1. The words “Year 10 pupil” restrict the group from which the selection is made.

### Worked Solution

$$n(Y10) = 30$$

$$n(Y10 \text{ bus}) = 12$$

$$P = 12/30 = 2/5$$

Use only the Year 10 row.

Divide the relevant cell by the row total.

## 3. Two-way tables

### Example Problem 4

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Group A has 12 bus users out of 30 people. Group B has 20 bus users out of 50. Is bus use independent of group?

### The idea

1. Compare the probability of bus use in each group. Equal probabilities mean the group does not change the chance of bus use.

### Worked Solution

$$P = 12/30 = 0.4$$

Calculate the first proportion.

$$P = 20/50 = 0.4$$

Calculate the second proportion.

Independent

Both groups have the same bus-use probability.

### 3. Two-way tables

#### Example Problem 5

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The table records 100 pupils. Among those who cycle, the probability of being in Year 10 is  $\frac{2}{5}$ . Find  $x$ .

|         | Cycle | Do not cycle |
|---------|-------|--------------|
| Year 10 | $x$   | 30           |
| Year 11 | 18    | $52 - x$     |

#### The idea

1. Restrict the denominator to pupils who cycle.
2. Write the probability using  $x$ , then multiply both sides by the denominators.

#### Worked Solution

$$\frac{x}{x + 18} = \frac{2}{5}$$

$$5x = 2(x + 18)$$

$$5x = 2x + 36 \Rightarrow x = 12$$

$$\frac{12}{12 + 18} = \frac{2}{5}$$

The cyclist column contains  $x + 18$  pupils.

Multiply both sides by  $5(x + 18)$ .

Subtract  $2x$ , then divide by 3.

Check the required probability.

**3 Two-way tables**

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**Q13** A pupil is chosen at random from the table. Find the probability that they travel by bus.

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|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 18   | 12  | 30    |
| Year 11 | 22   | 18  | 40    |
| Total   | 40   | 30  | 70    |

**Q14** A Year 11 pupil is chosen at random from the table. Find the probability they walk.

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|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 12   | 18  | 30    |
| Year 11 | 24   | 16  | 40    |
| Total   | 36   | 34  | 70    |

**Q15** A pupil who travels by bus is chosen at random from the table. Find the probability they are in Year 10.

\*\*\*

|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 10   | 20  | 30    |
| Year 11 | 30   | 10  | 40    |
| Total   | 40   | 30  | 70    |

**Q16** Complete the four missing values in the table.

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|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 25   | ?   | 40    |
| Year 11 | ?    | ?   | 50    |
| Total   | ?    | 37  | 90    |

**Q17** In the table, is travelling by bus independent of year group? Justify your answer.

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|         | Walk | Bus | Total |
|---------|------|-----|-------|
| Year 10 | 15   | 10  | 25    |
| Year 11 | 30   | 20  | 50    |
| Total   | 45   | 30  | 75    |

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**Q18** The table records 120 people. Among swimmers, the probability of being under 18 is  $\frac{3}{5}$ . Find  $x$ .

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|          | Swim | Do not swim |
|----------|------|-------------|
| Under 18 | $x$  | 40          |
| Adult    | 20   | $60 - x$    |

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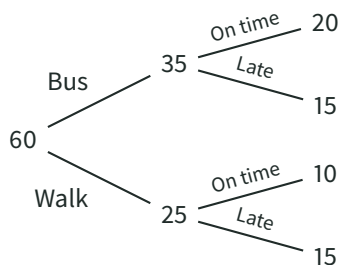
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## 4. Frequency trees and constraints

### Example Problem 6

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Using the frequency tree, find the probability that a person chosen from those on time took the bus.



### The idea

1. Combine matching endpoints before calculating a conditional probability.

### Worked Solution

$$n(\text{on time}) = 20 + 10 = 30$$

Add the two on-time endpoints.

$$P = 20/30 = 2/3$$

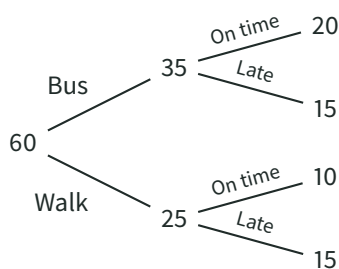
Use all on-time people as the denominator.

## 4 Frequency trees and constraints

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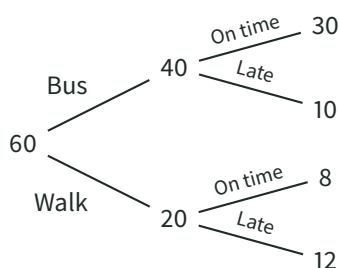
**Q19** Use the frequency tree to find how many people are late.

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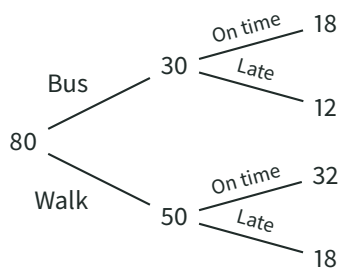
**Q20** Use the frequency tree to find the probability that a randomly selected person walks and is on time.

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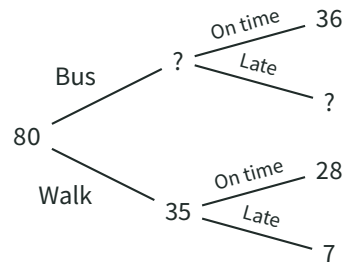
**Q21** Use the frequency tree. A late person is chosen at random. Find the probability they travel by bus.

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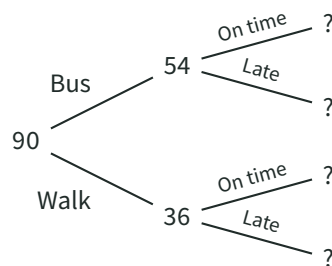
**Q22** Complete the missing counts in the frequency tree.

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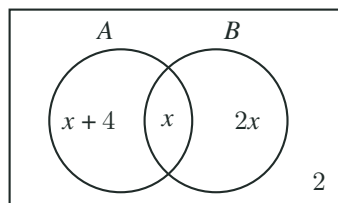
**Q23** A group has 90 people. Of the 54 bus passengers,  $\frac{5}{6}$  are on time. Of the walkers,  $\frac{3}{4}$  are on time. A late person is chosen at random. Find the probability they walk.

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**Q24** The Venn diagram represents a group. The probability of being in both sets is  $\frac{1}{5}$ . Find  $x$ .

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# Solving principles

## 1. Distinguish total and region

Being in A does not mean being in A only.

## 2. Check the whole population

The numbers in the separate regions of a Venn diagram, or cells of a table, must add to the total.

## 3. Choose the denominator

When someone is chosen from a particular group, use the number in that group as the denominator.

## 4. Use whole numbers for counts

A probability may be fractional, but a count of people must be a whole number.