

GCSE MATHS

Units, rates and compound measures

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Conversions

Convert all quantities to compatible units before using a formula. An area conversion squares the length factor; a volume conversion cubes it.

Speed

$\text{speed} = \text{distance} / \text{time}$. Average speed uses total distance divided by total elapsed time.

Density

$\text{density} = \text{mass} / \text{volume}$. For a mixture, add masses and volumes separately when the total volume is the sum of the two volumes.

Pressure

$\text{pressure} = \text{force} / \text{area}$. One pascal is one newton per square metre.

A calculator may be used. Give exact answers unless the question asks you to round.

1. Units and rates

Example Problem 1

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Convert 90 km/h to m/s.

The idea

1. Convert the numerator and denominator separately.

Worked Solution

$$90 \text{ km} = 90000 \text{ m}$$

$$1 \text{ h} = 3600 \text{ s}$$

$$v = 90000/3600 = 25 \text{ m/s}$$

Convert the distance travelled in an hour.

Convert the time.

Divide in the required units.

1 Units and rates

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Q1 Convert 72 km/h to m/s.

**

Q2 Convert 15 m/s to km/h.

**

Q3 Convert 350 cm² to m².

**

Q4 Convert 0.004 m³ to cm³.

**

Q5 Convert a density of 2.7 g/cm³ to kg/m³.

**

Q6 A tap delivers 18 litres in 45 seconds. Find its rate in litres per minute.

**

2. Speed and journeys

Example Problem 2

A car travels 30 km at 60 km/h and then 30 km at 40 km/h. Find its overall average speed.

The idea

1. Find the time for each part of the journey.
2. Divide the total distance by the total time. The car spends longer at 40 km/h, so its average speed is below 50 km/h.

Worked Solution

$$t_1 = \frac{30}{60} = 0.5 \text{ h}$$

Divide the first distance by its speed.

$$t_2 = \frac{30}{40} = 0.75 \text{ h}$$

The slower part takes longer.

$$v = \frac{30 + 30}{0.5 + 0.75} = 48 \text{ km/h}$$

Divide the total distance by the total time.

2 Speed and journeys

Q7 A car travels 156 km in 2 hours 24 minutes. Find its average speed.

**

Q8 A cyclist travels at 18 km/h for 35 minutes. Find the distance.

**

Q9 A train leaves at 09:42 and travels 168 km at an average speed of 96 km/h. When does it arrive?

**

Q10 A driver covers 40 km at 50 km/h, stops for 12 minutes, then covers 60 km at 75 km/h. Find the average speed for the whole journey, to 1 decimal place.

Q11 Two runners start 600 m apart and run towards each other at 4 m/s and 6 m/s. How long until they meet?

Q12 A cyclist passes a point at 6 m/s. A second cyclist passes it 20 seconds later at 8 m/s in the same direction. How long after passing the point does the second cyclist catch up?

3. Density and pressure

Example Problem 3

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A sample has mass 0.96 kg and density 1.2 g/cm^3 . Find its volume.

The idea

1. Rearrange the density formula and make the units agree.

Worked Solution

$$m = 960 \text{ g}$$

Match the mass units in the density.

$$V = m/\text{density} = 960/1.2 = 800 \text{ cm}^3$$

Divide mass by density.

3. Density and pressure

Example Problem 4

**

A 400 N force acts on 50 cm^2 . Find the pressure in pascals.

The idea

1. A pascal is a newton per square metre, so convert area before dividing.

Worked Solution

$$A = 50/10000 = 0.005 \text{ m}^2$$

Convert area to square metres.

$$P = 400/0.005 = 80000 \text{ Pa}$$

Divide force by area.

3 Density and pressure

Q13 A material has mass 540 g and volume 200 cm^3 . Find its density.

**

Q14 A liquid has density 0.8 g/cm^3 and volume 350 cm^3 . Find its mass.

**

Q15 A solid of density 7.5 g/cm^3 has mass 1.8 kg. Find its volume.

**

Q16 A force of 600 N acts on an area of 0.025 m^2 . Find the pressure in pascals.

**

Q17 A pressure of 180 kPa acts on an area of 0.04 m^2 . Find the force.

Q18 A force of 240 N produces a pressure of 30,000 Pa. Find the area in cm^2 .

4. Combining rates and measures

Example Problem 5

Mix 150 cm^3 of density 2 g/cm^3 with 50 cm^3 of density 6 g/cm^3 . Assume the total volume is the sum of the two volumes. Find the density.

The idea

1. The quantities mixed have unequal volumes, so their densities do not receive equal weight.

Worked Solution

$$m = 150(2) + 50(6) = 600 \text{ g}$$

Calculate both masses before adding.

$$V = 150 + 50 = 200 \text{ cm}^3$$

Add volumes.

$$\text{density} = 600/200 = 3 \text{ g/cm}^3$$

Divide the totals.

4. Combining rates and measures

Example Problem 6

A mixture contains 80 cm^3 of a liquid with density 3 g/cm^3 and $x \text{ cm}^3$ of a liquid with density 6 g/cm^3 . Assume the total volume is the sum of the two volumes. The mixture has density 4.2 g/cm^3 . Find x .

The idea

1. Write each mass as density multiplied by volume.
2. Use total mass = mixture density multiplied by total volume, then solve for x .

Worked Solution

$$m = 80(3) + 6x = 240 + 6x$$

Add the two masses, in grams.

$$V = 80 + x$$

Add the volumes, in cubic centimetres.

$$240 + 6x = 4.2(80 + x)$$

Multiply the mixture density by its volume.

$$240 + 6x = 336 + 4.2x$$

Expand the bracket.

$$1.8x = 96 \Rightarrow x = \frac{160}{3} \text{ cm}^3$$

Subtract $4.2x$ and 240, then divide by 1.8.

4 Combining rates and measures

★★★★

Q19 200 cm³ of liquid of density 0.8 g/cm³ is mixed with 300 cm³ of density 1.2 g/cm³. Assume the total volume is the sum of the two volumes. Find the mixture density.

★★★

Q20 A car travels 60 km outwards at 40 km/h and 60 km back at 60 km/h. Find its average speed over the whole journey.

★★★

Q21 A car must average 60 km/h over a 90 km journey. It covers the first 30 km at 40 km/h. What average speed is needed for the remaining distance?

★★★

Q22 Tap A fills a tank in 6 hours and tap B fills it in 4 hours. At constant rates, how long do they take together? Give hours and minutes.

Q23 A mixture contains 100 cm^3 of density 2 g/cm^3 and $x \text{ cm}^3$ of density 5 g/cm^3 . Assume the total volume is the sum of the two volumes. Its density is 3.2 g/cm^3 . Find x .

Q24 The force exerted by a machine stays the same and its contact area falls by 20%. By what percentage does the pressure increase?

Solving principles

1. Write the units first

Choose the distance, time, mass, volume or area units required by the answer.

2. Use totals

Combine distances, times, masses or volumes before dividing to find an overall rate.

3. Distinguish time from clock time

Subtract departure from arrival, including any change of hour.

4. Build an equation

When an unknown occurs in a total or in two rates, write both contributions using the same unknown.