

GCSE MATHS

Trigonometric graphs and transformations

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Degrees

Use degrees for these graphs. Sine and cosine repeat every 360° ; tangent repeats every 180° .

Key values

$\sin 0^\circ = 0$, $\sin 90^\circ = 1$, $\cos 0^\circ = 1$ and $\cos 90^\circ = 0$. Tangent is undefined at 90° , 270° and at intervals of 180° in either direction.

Translations

$y = f(x) + a$ moves a graph up a . $y = f(x - a)$ moves it right a : the same function input occurs a units later.

Reflections

$y = -f(x)$ reflects in the x -axis. $y = f(-x)$ reflects in the y -axis.

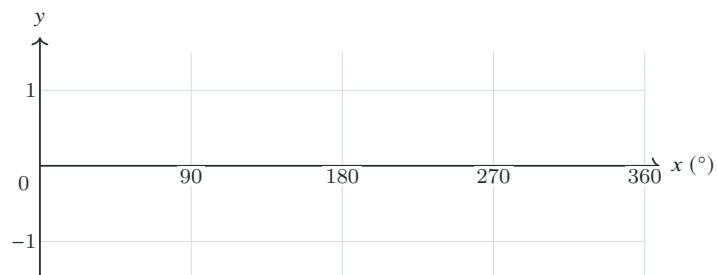
Use degree mode. Give angles to the nearest degree unless an exact value is requested.

1. Sine and cosine graphs

Example Problem 1

★ ★

Sketch $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$.



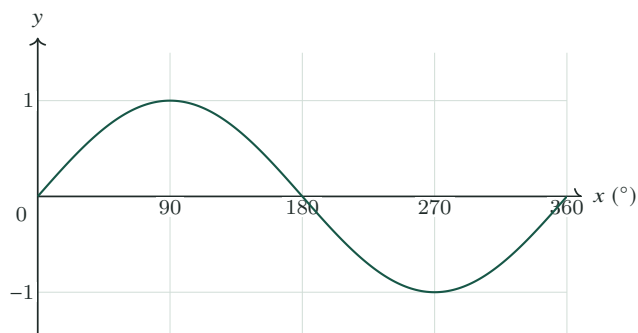
The idea

1. Mark the five quarter-turn points, then join smoothly.

Worked Solution

$(0, 0), (90, 1), (180, 0), (270, -1), (360, 0)$

The graph completes one cycle.

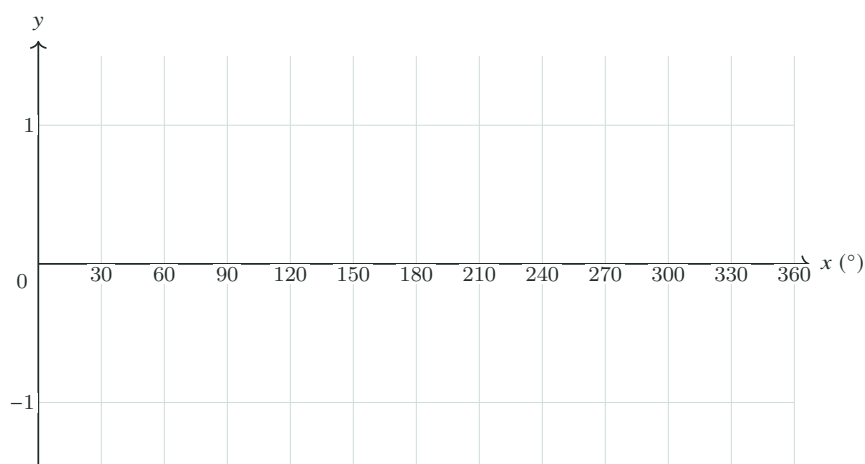


1 Sine and cosine graphs

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Q1 Write down the value of $\sin x$ for each angle: 0° , 90° , 180° , 270° , 360° . Sketch the graph on the axes.

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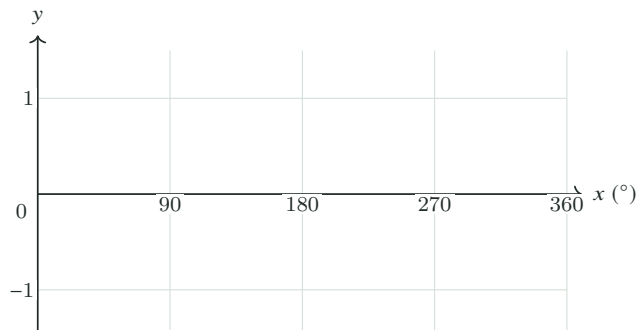


Q2 Consider $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$.

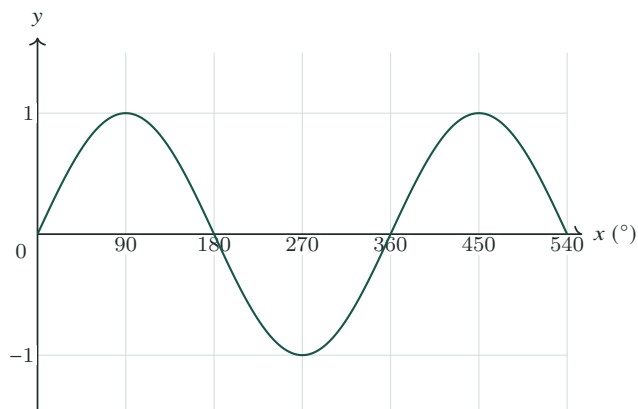
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(a) Write down $\cos x$ for $x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$.

(b) Sketch the graph, marking these five points.



Q3 The graph shows $y = \sin x$. Write down the solutions of $\sin x = 0$ for $0^\circ < x < 540^\circ$. **



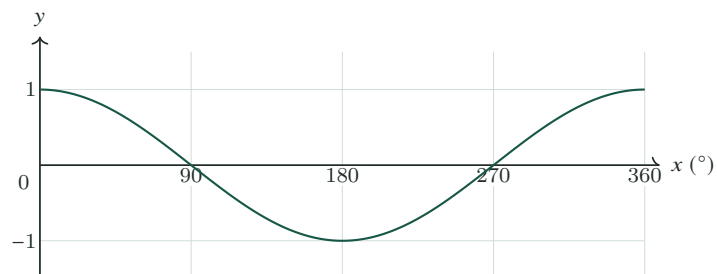
Q4 For $0^\circ \leq x \leq 360^\circ$, find (a) the minimum value of $\sin x$; (b) the value of x where it occurs. **

Q5 For $0^\circ \leq x \leq 360^\circ$, solve $\cos x = -1$. **

2. Tangent and graphical equations

Example Problem 2

Solve $\cos x = 0.5$ for $0^\circ \leq x \leq 360^\circ$.



The idea

1. Find the first angle with inverse cosine, then use the symmetry of the graph.

Worked Solution

$$x = \cos^{-1}(0.5) = 60^\circ$$

First intersection.

$$x = 360^\circ - 60^\circ = 300^\circ$$

Second intersection.

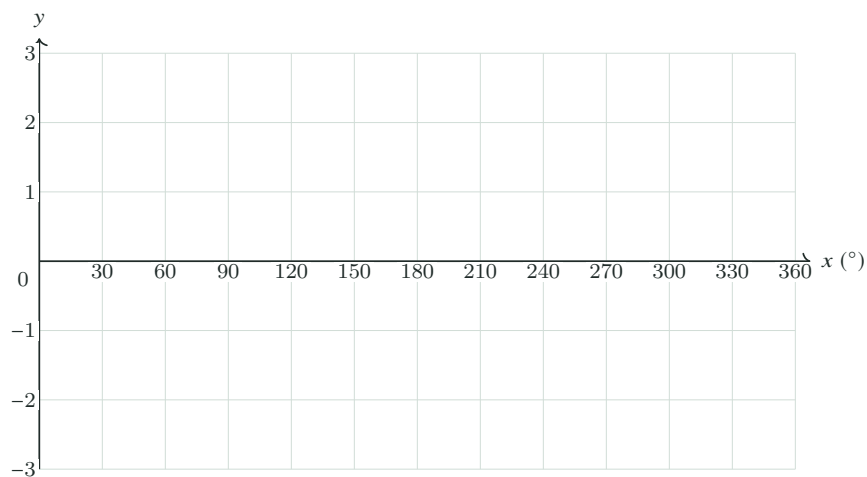
2 Tangent and graphical equations

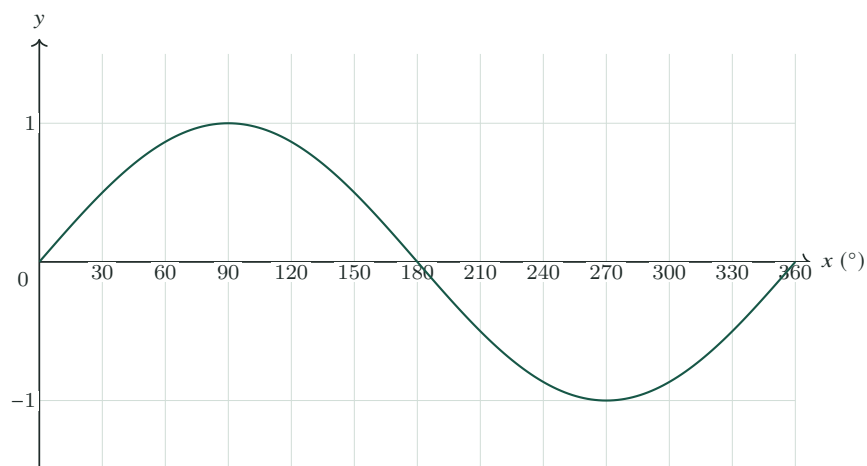
Q6 Find the exact value of $\tan 45^\circ$.

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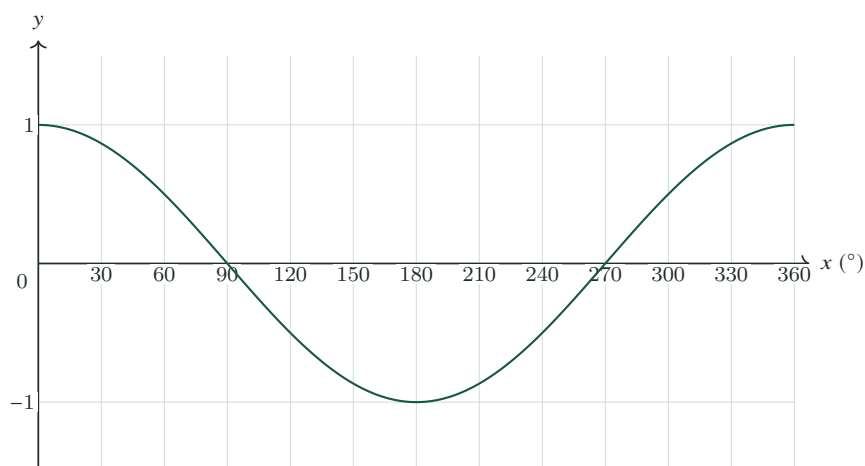
Q7 Sketch $y = \tan x$ for $0^\circ \leq x \leq 360^\circ$. Draw its vertical asymptotes as dashed lines.

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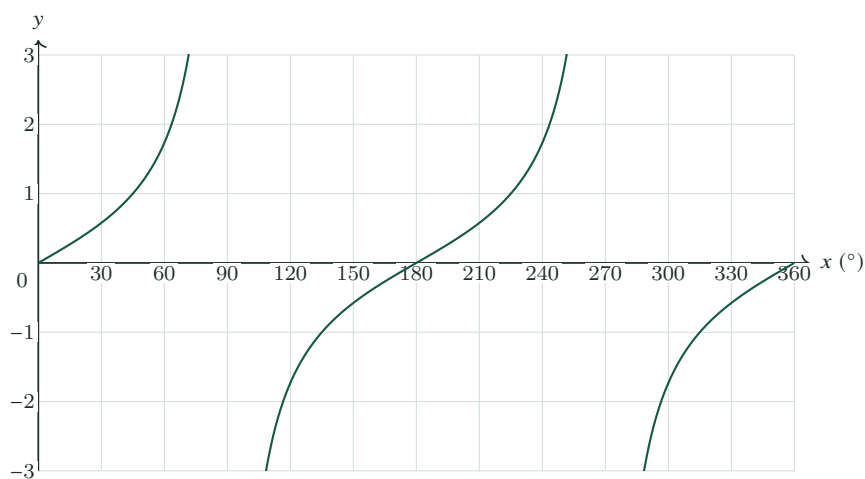
**Q8** Solve $\sin x = 0.5$ for $0^\circ \leq x \leq 360^\circ$.



Q9 Solve $\cos x = -0.5$ for $0^\circ \leq x \leq 360^\circ$.



Q10 Solve $\tan x = 1$ for $0^\circ \leq x \leq 360^\circ$.



Q11 Solve $\sin x = 0.7$ for $0^\circ \leq x \leq 360^\circ$.

3. Translating graphs

Example Problem 3

A graph $y = f(x)$ contains $(2, 5)$. Find the corresponding point on $y = f(x - 3) + 1$.

The idea

1. The input becomes 2 when x is 5, so move right three. Then add one to y .

Worked Solution

$$x - 3 = 2 \Rightarrow x = 5$$

Match the original input.

$$y = 5 + 1 = 6$$

Apply the vertical shift.

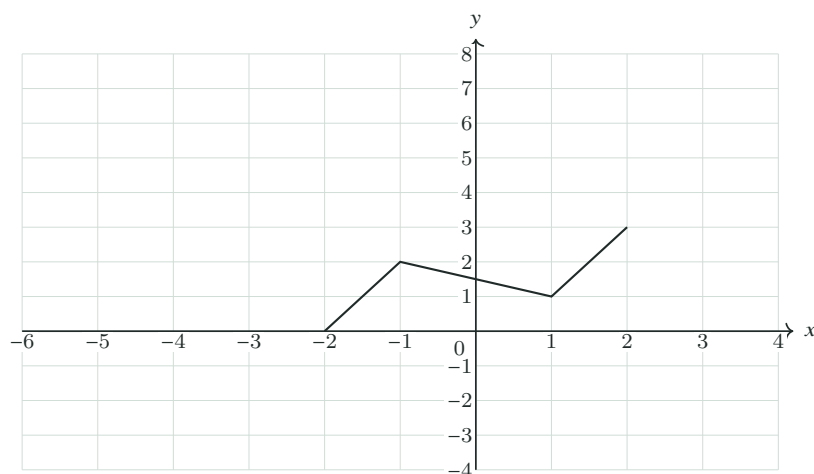
$$(5, 6)$$

State the new point.

3 Translating graphs

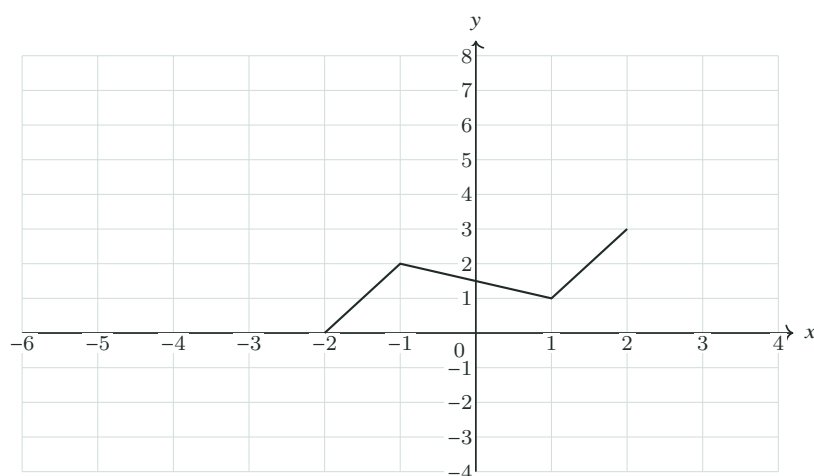
Q12 The graph shows $y = f(x)$. Draw $y = f(x) + 4$ on the same axes. Describe the transformation.

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Q13 The graph shows $y = f(x)$. Draw $y = f(x + 3)$ on the same axes. Describe the transformation.

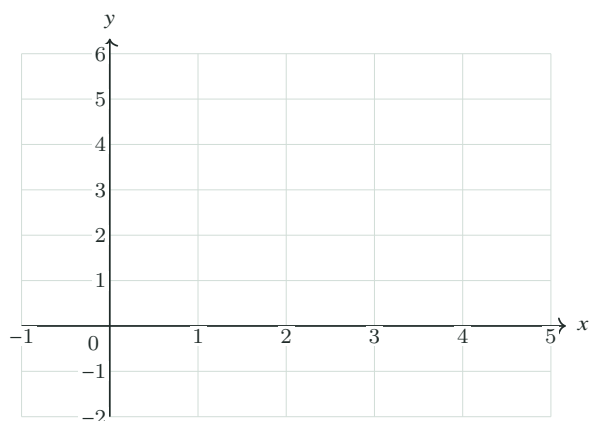
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Q14 The minimum of $y = f(x)$ is $(2, -3)$. Find the minimum of $y = f(x - 4) + 2$.

Q15 The graph $y = f(x)$ contains $(-1, 5)$. Find the corresponding point on $y = f(x + 2) - 3$.

Q16 Sketch $y = (x - 2)^2 - 1$. Label the turning point and roots.



Q17 Translating $y = f(x)$ moves $(1, 4)$ to $(5, 2)$. Write the new equation in terms of f .

4. Reflecting graphs and combining moves

Example Problem 4

A graph $y = f(x)$ contains $(-2, 3)$. Find the corresponding point on $y = -f(-x)$.

The idea

1. Reflect in the y -axis, then in the x -axis.

Worked Solution

$$(-2, 3) \longrightarrow (2, 3)$$

Reverse the x -coordinate.

$$(2, 3) \longrightarrow (2, -3)$$

Reverse the y -coordinate.

4. Reflecting graphs and combining moves

Example Problem 5

The graph $y = f(x)$ has a maximum at $(3, 2)$. Find the corresponding turning point on $y = -f(1 - x) + 4$.

The idea

1. Match the old input 3, then transform its output 2.

Worked Solution

$$1 - x = 3 \Rightarrow x = -2$$

Find the new horizontal position.

$$y = -2 + 4 = 2$$

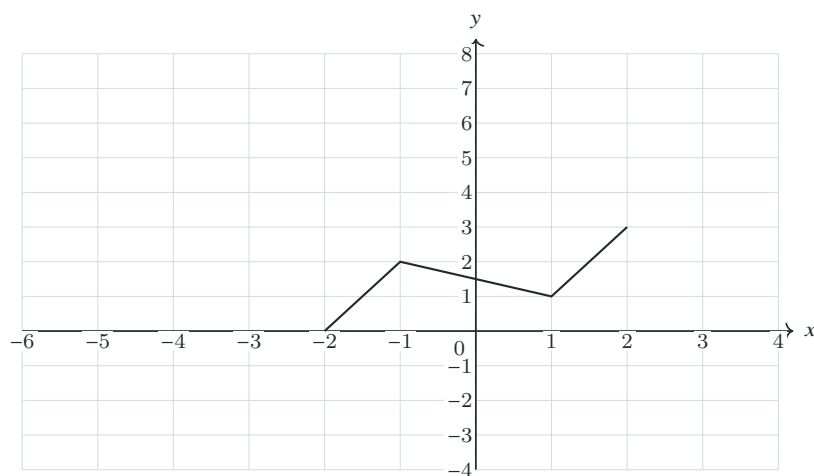
Reflect the output and shift it up.

Minimum at $(-2, 2)$

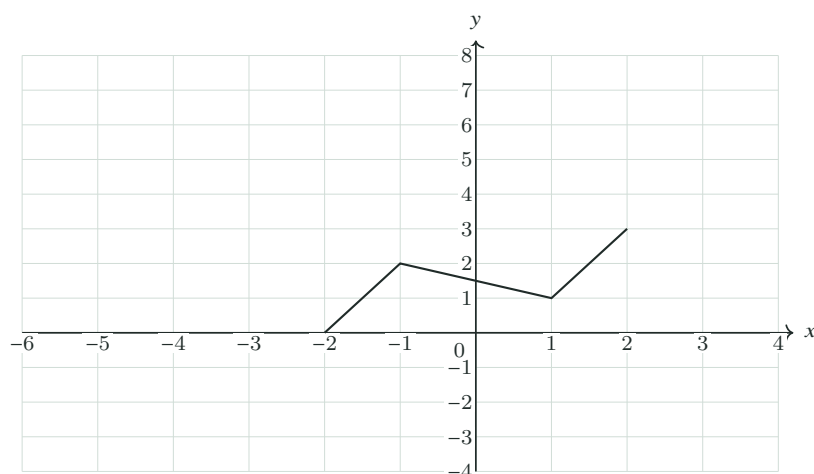
The vertical reflection swaps maximum and minimum.

4 Reflecting graphs and combining moves

Q18 The graph shows $y = f(x)$. Draw $y = -f(x)$ on the same axes. Describe the transformation. **



Q19 The graph shows $y = f(x)$. Draw $y = f(-x)$ on the same axes. Describe the transformation. **



Q20 The minimum of $y = f(x)$ is $(-2, 1)$. Find the maximum of $y = -f(x) + 5$. ***

Q21 If $(4, -3)$ lies on $y = f(x)$, find the corresponding point on $y = -f(-x) + 2$.

Q22 $y = f(x)$ has roots -1 and 3 . Find the roots of $y = f(2 - x)$.

Q23 For $0^\circ \leq x \leq 360^\circ$, solve $1 - \cos x = 1.5$.

Solving principles

1. Mark familiar points

Use zeros, maxima, minima and asymptotes before adding detail.

2. Use the full interval

Find every solution in the stated range, including endpoints.

3. Follow a point

Track one labelled point to check the direction of a transformation.

4. Keep the input separate

An outside change alters y ; an inside change alters the x -value that supplies the input.