

GCSE MATHS

Systematic listing and counting

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Systematic lists and factor pairs	Q1–6	**	3
2. The product rule	Q7–12	**	5
3. Choosing and arranging objects	Q13–18	***	7
4. Counting with restrictions	Q19–24	****	9
Solving principles			12

Topic recap

Make an organised list

To make two-digit numbers from 1, 2 and 3 without repeating a digit, list those beginning with 1, then 2, then 3:
12, 13; 21, 23; 31, 32.

Multiply the choices

With 3 drinks and 4 sandwiches, there are $3 \times 4 = 12$ meal choices. Each drink can be paired with each sandwich.

Check whether repeats are allowed

A three-digit code using 1, 2, 3 and 4 has $4 \times 4 \times 4$ possibilities if digits can repeat, or $4 \times 3 \times 2$ if they cannot.

Count each choice once

Choosing Ali and Bea is the same pair as choosing Bea and Ali. When listing pairs, do not count both orders.

1. Systematic lists and factor pairs

Example Problem 1

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List the two-digit numbers made from 1, 3 and 5 without repeating a digit.

The idea

1. Fix the tens digit in turn, then list each permitted units digit.

Worked Solution

13, 15; 31, 35; 51, 53

Three groups, each containing two outcomes.

$$3 \times 2 = 6$$

Check the list length.

1 Systematic lists and factor pairs

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Q1 List all positive factor pairs of 18.

*

Q2 List all pairs of positive integers with sum 9. Treat reversed pairs as the same pair.

**

Q3 List the two-digit even numbers made from 1, 2 and 4 without repeated digits.

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Q4 In how many ways can 24 be written as a product of two positive integers, if 2×12 and 12×2 count as different?

**

Q5 A game uses one red die and one blue die. How many outcomes have total 5?

**

Q6 A cafe sells small, medium and large drinks. Each size comes in tea, coffee or hot chocolate. List all the choices. How many are there?

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2. The product rule

Example Problem 2

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A meal has a choice of 3 starters, 4 mains and 2 desserts. How many different meals are possible?

The idea

1. Choose exactly one item from each course.

Worked Solution

$$3 \times 4 \times 2 = 24$$

Each starter can be paired with every main and dessert.

2 The product rule

**

Q7 Choose one of 5 drinks and one of 7 sandwiches. How many choices are possible?

*

Q8 A uniform has 3 shirt choices, 2 trouser choices and 4 tie choices. How many uniforms are possible?

**

Q9 A code consists of two letters chosen from A , B , C and D , followed by one digit from 0 to 9. Letters may repeat. How many codes are possible?

**

Q10 There are 6 routes from A to B and 3 routes from B to C . How many A to C journeys via B are possible?

**

Q11 A shop sells 4 sizes of a shirt in several colours. There are 28 size-colour combinations. How many colours are there?

**

Q12 A four-digit PIN uses 0 to 9, where digits may repeat and the first digit may be zero. How many PINs are possible?

**

3. Choosing and arranging objects

Example Problem 3

Five pupils each shake hands with every other pupil once. How many handshakes occur?

The idea

1. Count partners for every pupil, then remove the double counting.

Worked Solution

$$5 \times 4 = 20$$

Each handshake has been counted from both ends.

$$20/2 = 10$$

Count each pair once, regardless of its order.

3 Choosing and arranging objects

Q13 How many three-letter codes can be made from A, B, C, D and E without repeating a letter?

**

Q14 Six runners compete for first, second and third place. There are no ties. How many ways of finishing first, second and third are possible?

Q15 How many different pairs can be chosen from 8 pupils? The order of the pair does not matter.

Q16 A league has 7 teams. Each pair plays once. How many matches are played?

Q17 Four different books stand in a row. In how many orders can they be arranged?

**

Q18 A committee needs a chair and a secretary chosen from 9 pupils. The same pupil cannot do both jobs. In how many ways can the two pupils be chosen?

4. Counting with restrictions

Example Problem 4

A three-digit code uses digits 0 to 4. Digits cannot repeat. How many codes contain 0? The first digit may be zero.

The idea

1. Subtract codes containing no zero from all permitted codes.

Worked Solution

$$5 \times 4 \times 3 = 60$$

Count all codes without repetition.

$$4 \times 3 \times 2 = 24$$

Count codes that use only 1 to 4.

$$60 - 24 = 36$$

Keep codes containing zero.

4 Counting with restrictions

★★★★

Q19 How many three-digit numbers use only 1, 2, 3 and 4, without repetition, and are even?

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Q20 How many three-digit numbers use 0, 1, 2, 3 and 4 without repetition?

★★★

Q21 A three-digit PIN uses 0 to 9, where digits may repeat and the first digit may be zero. How many PINs contain at least one 7?

★★★★

Q22 A three-digit number uses the digits 1, 2, 3, 4 and 5 without repetition. The number is a multiple of 5. How many such numbers are there? ***

Q23 Choose two pupils from a group of 4 girls and 3 boys. How many pairs contain at least one girl? ****

Q24 How many three-digit numbers greater than 300 can be made from 0, 1, 2, 3 and 4 without repetition, if the number is even? ****

Solving principles

1. Define an outcome

Distinguish a code from a selection of people: order may matter in one and not the other.

2. Check restrictions first

Check whether the first digit can be zero, whether digits can repeat and which choices are allowed.

3. Avoid double counting

If two orders describe the same selection, count it once.

4. Check small cases

A short organised list can test a counting rule.