

GCSE MATHS

Straight-line graphs

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Gradient

$m = (y_2 - y_1)/(x_2 - x_1)$ is the change in y divided by the change in x . Keep the subtraction order consistent. A vertical line has an undefined gradient.

Line equation

In $y = mx + c$, m is the gradient and $(0, c)$ is the y -intercept. Substitute a known point to find c .

Parallel and perpendicular

Parallel non-vertical lines have equal gradients. Perpendicular non-horizontal lines have gradients whose product is -1 . Horizontal and vertical lines are perpendicular.

Intersections

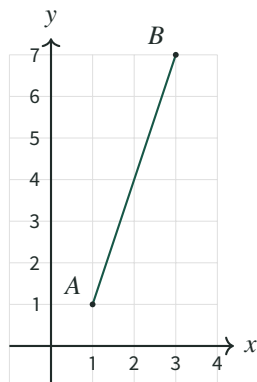
At an intersection, the coordinates satisfy both equations. Equate the expressions for y , then substitute the x -value into either line.

1. Gradients and intercepts

Example Problem 1

★

Find the gradient between (1, 1) and (3, 7).



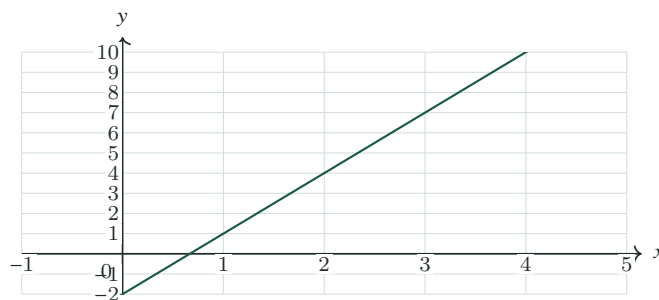
The idea

1. Choose the same point first in both subtractions.
2. Gradient is vertical change divided by horizontal change.

Worked Solution

$$m = \frac{7 - 1}{3 - 1} = 3$$

Divide vertical change by horizontal change.

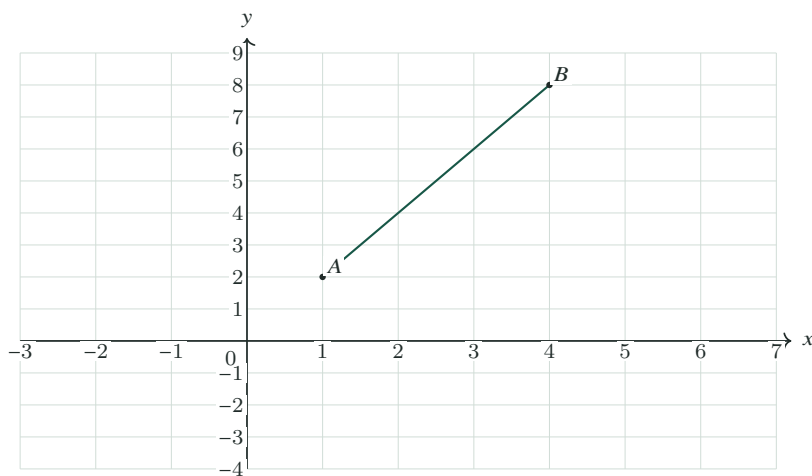


1 Gradients and intercepts

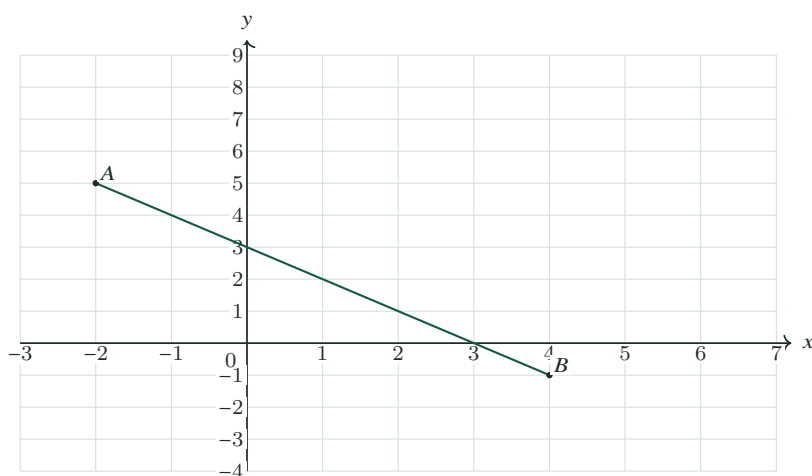
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Q1 Find the gradient of the line through $(1, 2)$ and $(4, 8)$.

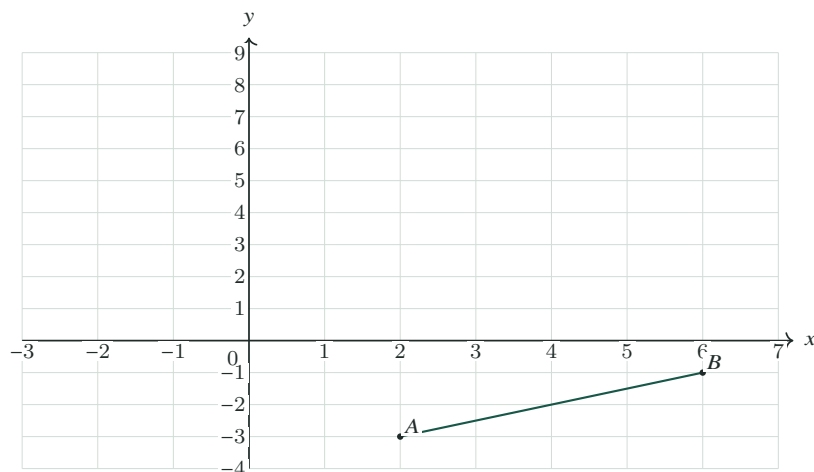
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**Q2** Find the gradient of the line through $(-2, 5)$ and $(4, -1)$.

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Q3 Find the gradient of the line through $(2, -3)$ and $(6, -1)$. ★

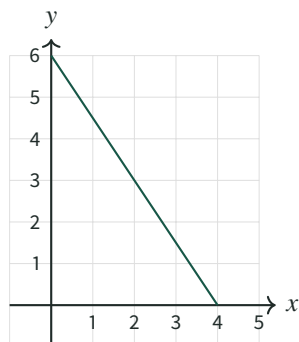


Q4 State the gradient and y-intercept of $y = 4 - 3x$. Sketch the graph on the axes. ★



Q5 For $3x + 2y = 12$, find (a) the x -intercept; (b) the y -intercept.

**



Q6 Explain why the line through $(3, -2)$ and $(3, 7)$ cannot be written as $y = mx + c$ with a numerical value for the gradient. State its equation.

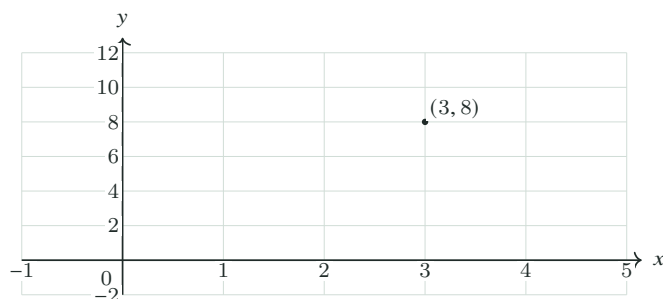
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2. Finding line equations

Example Problem 2

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Find the equation of the line of gradient 2 through $(3, 8)$.



The idea

1. The gradient fixes the coefficient of x .
2. Use the point to find the remaining constant.

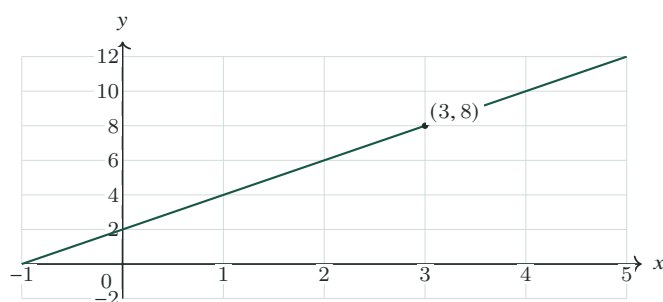
Worked Solution

$$8 = 2(3) + c$$

Substitute into $y = mx + c$.

$$c = 2 \Rightarrow y = 2x + 2$$

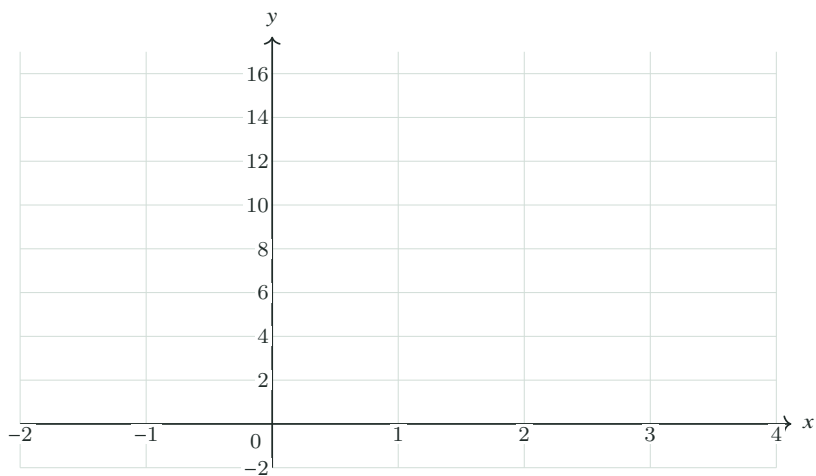
Find the intercept.



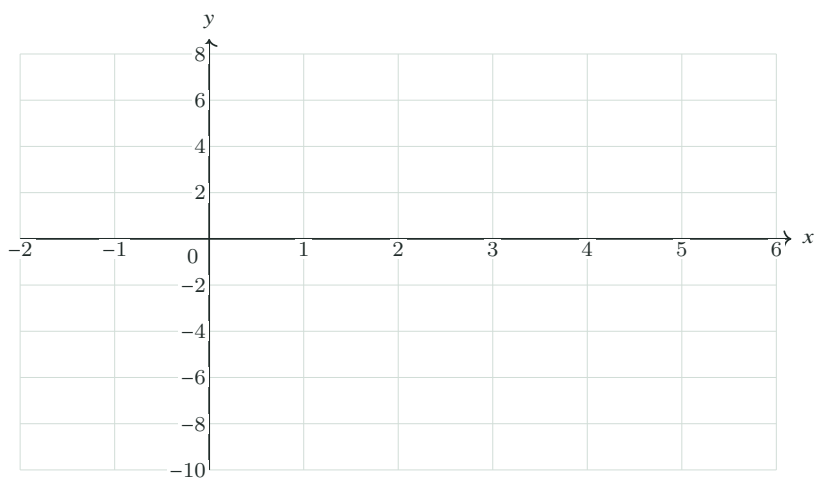
2 Finding line equations

Q7 Find the equation of the line with gradient 3 through $(2, 11)$. Sketch the graph on the axes.

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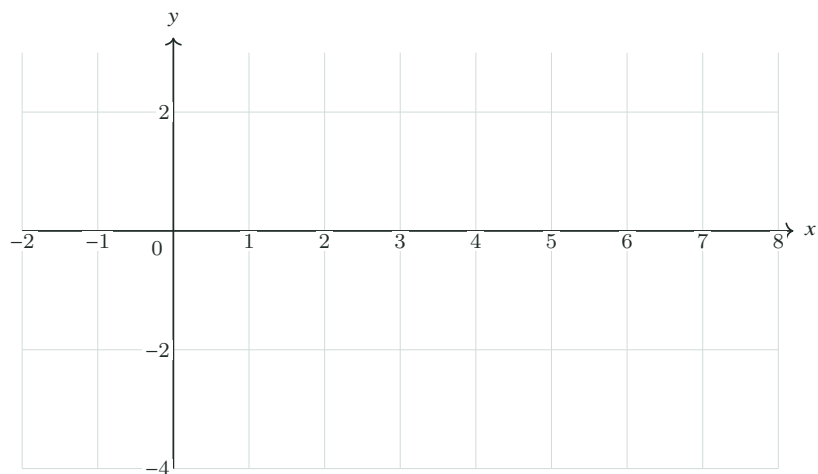
**Q8** Find the equation of the line with gradient -2 through $(4, -5)$. Sketch the graph on the axes.

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Q9 Find the equation of the line with gradient 0.5 through (6, 1). Sketch the graph on the axes.

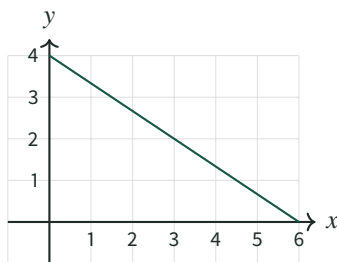
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Q10 Find the equation of the line through (1, 5) and (4, 14).

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Q11 Find the equation of the line with x -intercept 6 and y -intercept 4. **

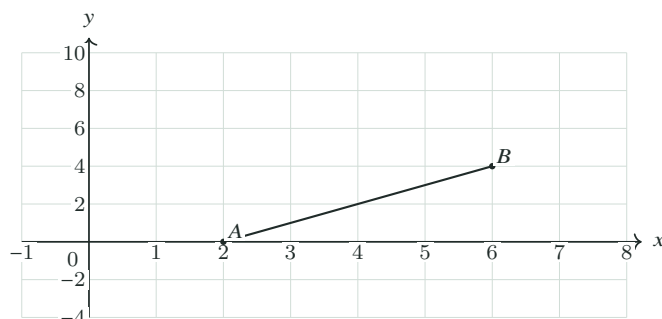


Q12 $A = (-1, 7)$, $B = (2, 1)$ and $C = (5, k)$ lie on the same straight line. Find k . ***

3. Parallel and perpendicular lines

Example Problem 3

Find the equation of the perpendicular bisector of the segment from $(2, 0)$ to $(6, 4)$.



The idea

1. A bisector passes through the midpoint.
2. Its gradient must be perpendicular to the segment.

Worked Solution

$$M = (4, 2)$$

Find midpoint and original gradient.

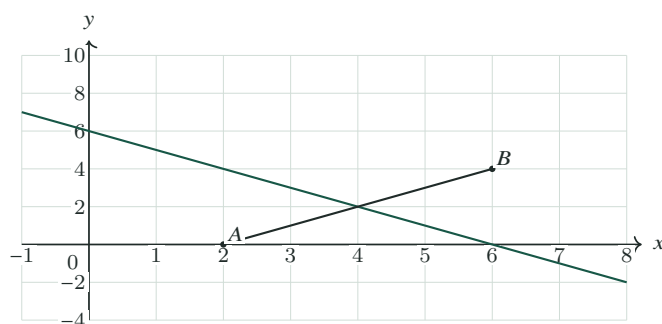
$$m = 1$$

Use the negative reciprocal.

$$m_{\perp} = -1$$

Pass the line through the midpoint.

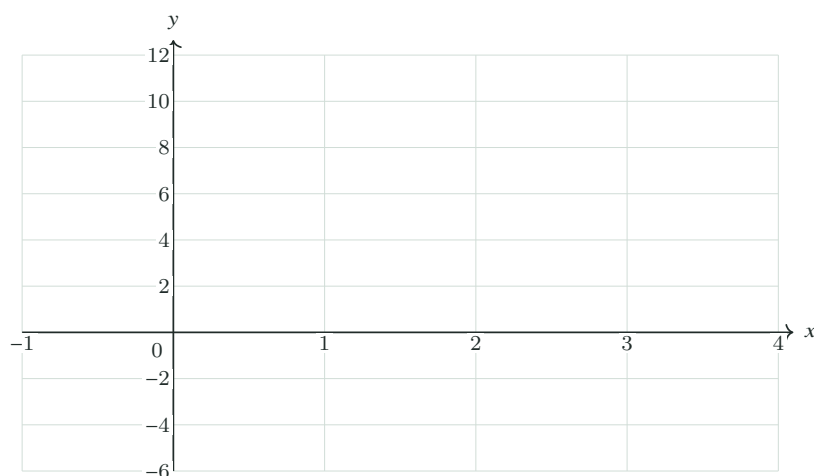
$$2 = -4 + c \Rightarrow y = -x + 6$$



3 Parallel and perpendicular lines

Q13 Find the equation of the line parallel to $y = 4x - 9$ passing through $(2, 3)$. Sketch the graph on the axes.

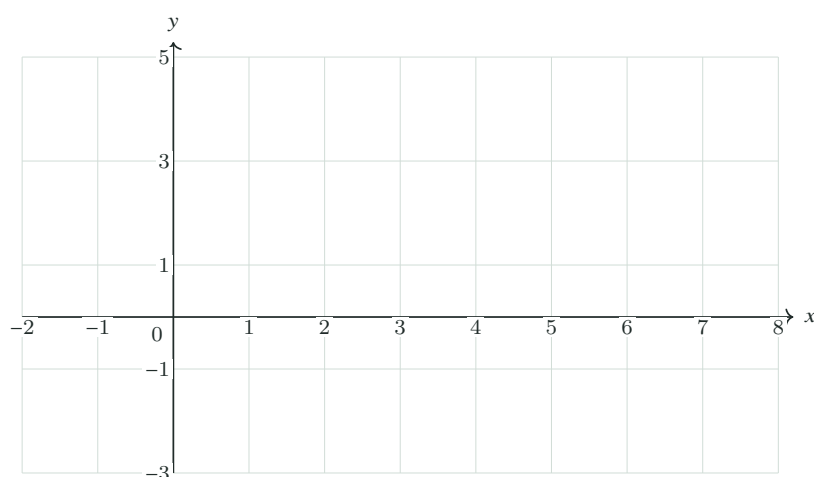
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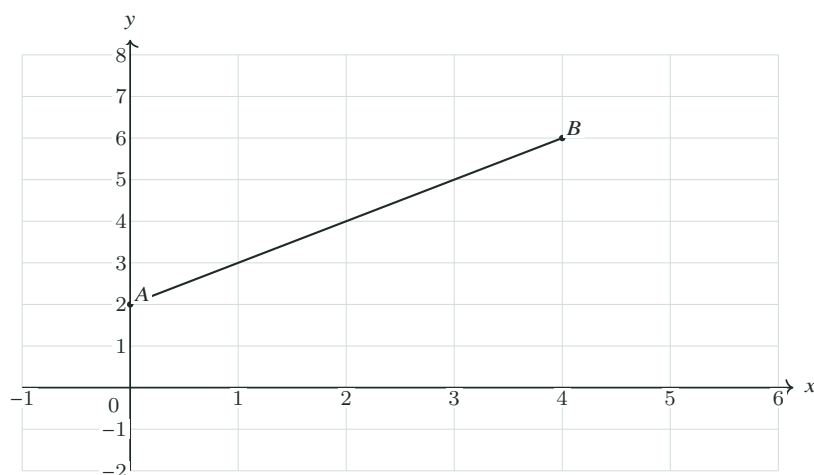
Q14 Find the gradient of a line perpendicular to $y = \frac{2}{3}x + 7$.

**

Q15 Find the equation of the line perpendicular to $y = -2x + 5$ passing through $(4, 1)$. Sketch the graph on the axes.



- Q16** Find the equation of the perpendicular bisector of the segment joining $(0, 2)$ and $(4, 6)$.
Sketch the graph on the axes. ***



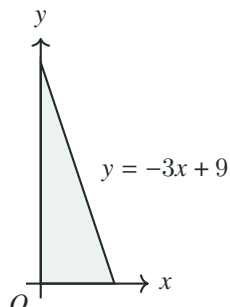
- Q17** The lines $2y = 3x + 4$ and $ky = 2x - 5$ are perpendicular. Find k . ***

- Q18** A line is perpendicular to $x = 5$ and passes through $(-2, 7)$. State its equation. ***

4. Intersections and coordinate geometry

Example Problem 4

Find the area between $y = -3x + 9$ and the positive coordinate axes.



The idea

1. Set one coordinate to zero at each axis.
2. Use the right triangle formed by the intercepts.

Worked Solution

$$x = 0 \Rightarrow y = 9$$

$$y = 0 \Rightarrow x = 3$$

$$A = \frac{1}{2}(3)(9) = 13.5$$

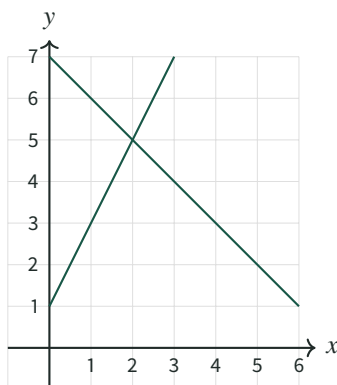
Find the intercepts.

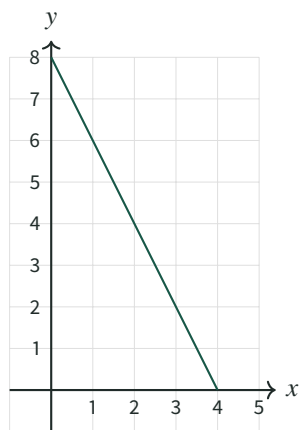
Use them as perpendicular base and height.

4 Intersections and coordinate geometry

Q19 Find the intersection of $y = 2x + 1$ and $y = -x + 7$.

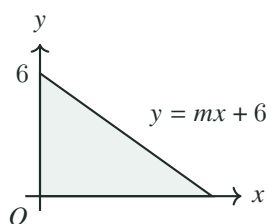
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**Q20** The line $y = -2x + 8$ and the coordinate axes enclose a triangle. Find its area.



Q21 The lines $y = x + 2$ and $y = 3x - 2$ meet at P . Find the distance from P to the origin.

Q22 The line $y = mx + 6$, with $m < 0$, and the positive coordinate axes enclose a triangle of area 9. Find m .



Q23 $A = (1, 1)$ and $B = (5, 3)$. Find the point on the y -axis equidistant from A and B .

Q24 The line $y = 2x + c$ meets the perpendicular line $y = -\frac{1}{2}x + 5$ on the x -axis. Find c .

Solving principles

1. Use coordinates consistently

A negative rise gives a negative gradient when the run is positive.

2. Check two points

Substitute the given coordinates into your finished line equation.

3. Distinguish x - and y -intercepts

Set $y = 0$ for the x -intercept; set $x = 0$ for the y -intercept.

4. Connect algebra to the diagram

Subtract coordinates to find horizontal or vertical lengths. A length must be positive.