

GCSE MATHS

Sine and cosine rules

Worked Solutions

Q1

**

$$b = \frac{8 \sin 65^\circ}{\sin 40^\circ}$$

$$b = 11.3 \text{ cm (1 d.p.)}$$

Match the two opposite pairs.

Evaluate without rounding the sine values.

Q2

**

$$b = \frac{12 \sin 35^\circ}{\sin 75^\circ}$$

$$b = 7.1 \text{ cm (1 d.p.)}$$

Match the two opposite pairs.

Evaluate without rounding the sine values.

Q3

**

$$b = \frac{6 \sin 45^\circ}{\sin 30^\circ}$$

$$b = 8.5 \text{ cm (1 d.p.)}$$

Match the two opposite pairs.

Evaluate without rounding the sine values.

Q4

**

$$\sin B = \frac{7 \sin 50^\circ}{10}$$

$$B = 32.4^\circ \text{ (1 d.p.)}$$

Rearrange the sine rule.

Use inverse sine; B is specified as acute.**Q5**

**

$$\sin B = \frac{6 \sin 65^\circ}{9}$$

$$B = 37.2^\circ \text{ (1 d.p.)}$$

Rearrange the sine rule.

Use inverse sine; B is specified as acute.**Q6**

$$C = 180 - 45 - 60 = 75^\circ$$

$$c = \frac{10 \sin 75^\circ}{\sin 60^\circ}$$

$$c = 11.2 \text{ cm (1 d.p.)}$$

Find the missing angle first.

Use the known opposite pair.

Evaluate.

Q7

**

$$a^2 = 7^2 + 9^2 - 2(7)(9) \cos 60^\circ$$

Use the included angle.

$$a = 8.2 \text{ cm (1 d.p.)}$$

Take the positive square root.

Q8

**

$$a^2 = 8^2 + 11^2 - 2(8)(11) \cos 110^\circ$$

Use the included angle.

$$a = 15.7 \text{ cm (1 d.p.)}$$

Take the positive square root.

Q9

**

$$a^2 = 5^2 + 8^2 - 2(5)(8) \cos 35^\circ$$

Use the included angle.

$$a = 4.8 \text{ cm (1 d.p.)}$$

Take the positive square root.

Q10

$$\cos A = \frac{7^2 + 8^2 - 9^2}{2(7)(8)}$$

Rearrange the cosine rule.

$$A = 73.4^\circ \text{ (1 d.p.)}$$

Use inverse cosine.

Q11

$$\cos A = \frac{7^2 + 9^2 - 13^2}{2(7)(9)}$$

Rearrange the cosine rule.

$$A = 108.0^\circ \text{ (1 d.p.)}$$

Use inverse cosine.

Q12

$$\cos A = \frac{8^2 + 10^2 - 6^2}{2(8)(10)}$$

Rearrange the cosine rule.

$$A = 36.9^\circ \text{ (1 d.p.)}$$

Use inverse cosine.

Q13

**

$$K = \frac{1}{2}(8)(11) \sin 40^\circ$$

Use the included angle in the area formula.

$$K = 28.3 \text{ cm}^2 \text{ (1 d.p.)}$$

Evaluate.

Q14

**

$$K = \frac{1}{2}(10)(6) \sin 120^\circ$$

Use the included angle in the area formula.

$$K = 26.0 \text{ cm}^2 \text{ (1 d.p.)}$$

Evaluate.

Q15

**

$$K = \frac{1}{2}(12)(7) \sin 30^\circ$$

Use the included angle in the area formula.

$$K = 21.0 \text{ cm}^2 \text{ (1 d.p.)}$$

Evaluate.

Q16

$$24 = \frac{1}{2}(8)(12) \sin A$$

Use the area formula.

$$\sin A = \frac{1}{2}$$

Divide by 48.

$$A = 30^\circ \text{ or } 150^\circ$$

Both angles give positive valid triangles.

Q17

$$30 = \frac{1}{2}(x)(10) \sin 30^\circ$$

Use the two enclosing sides.

$$30 = 2.5x \Rightarrow x = 12 \text{ cm}$$

Use $\sin 30^\circ = 1/2$ and solve.**Q18**

$$K = 20 \sin A$$

Write area in terms of the included angle.

$$\sin A \leq 1 \Rightarrow K \leq 20$$

Sine cannot exceed 1.

$$A = 90^\circ \Rightarrow K = 20 \text{ cm}^2$$

This angle attains the bound.

Q19

$$\cos A = \frac{7^2 + 8^2 - 9^2}{2(7)(8)} = \frac{2}{7}$$

$$K = \frac{1}{2}(7)(8) \sin A$$

$$K = 26.8 \text{ cm}^2 \text{ (1 d.p.)}$$

Find the angle between sides 7 and 8.

Use the unrounded angle.

Evaluate.

Q20

$$a^2 = 6^2 + 10^2 - 2(6)(10) \cos 60^\circ = 76$$

$$a = 2\sqrt{19}$$

$$P = 16 + 2\sqrt{19} \text{ cm}$$

Use the cosine rule.

Simplify the root.

Add the two given sides.

Q21

$$(x + 4)^2 = x^2 + (x + 2)^2 + x(x + 2)$$

$$2x^2 - 2x - 12 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3$$

Use $\cos 120^\circ = -1/2$ in the cosine rule.

Expand and collect.

Divide by two and factorise.

Reject the negative length -2 .

Q22

$$BC = \sqrt{8^2 + 11^2 - 2(8)(11) \cos 50^\circ}$$

$$\text{saving} = 19 - BC = 10.5 \text{ km (1 d.p.)}$$

Find the direct distance.

Compare with the two-path distance.

Q23

$$20 \sin A = 10\sqrt{3} \Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

$$A = 60^\circ \text{ or } 120^\circ$$

$$a^2 = 89 - 80 \cos A = 49 \text{ or } 129$$

$$a = 7 \text{ or } \sqrt{129} \text{ cm}$$

Use the area to find sine.

Keep both possible angles.

Apply the cosine rule for each angle.

Take positive roots.

Q24

$$\sin B = \frac{7 \sin 30^\circ}{5} = 0.7$$

$$B = 44.4^\circ \text{ or } 135.6^\circ$$

$$30^\circ + B < 180^\circ \text{ in both cases}$$

Use the known opposite pair.

Consider the inverse-sine result and its supplement.

Both leave a positive third angle.