

GCSE MATHS

Sine and cosine rules

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. The sine rule	Q1–6	***	3
2. The cosine rule	Q7–12	***	6
3. Triangle areas	Q13–18	***	9
4. Combining methods	Q19–24	****	12
Solving principles			16

Topic recap

Opposite pairs

Use a opposite angle A , b opposite B , and c opposite C . Match a side to its opposite angle before substituting.

Sine rule

$a/\sin A = b/\sin B$. Use a known opposite pair. When finding an angle, the supplementary angle may also fit; use the given information to decide.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$. This connects two sides and their included angle, or three sides and an unknown angle.

Area

Area = $\frac{1}{2}bc \sin A$, where A is the included angle between b and c .

Use a calculator in degree mode.

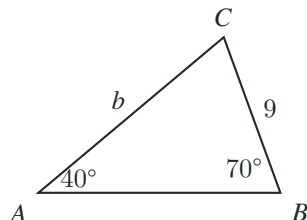
Calculate from the given information; do not measure the diagrams.

1. The sine rule

Example Problem 1

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$a = 9$ cm, $A = 40^\circ$ and $B = 70^\circ$. Find b to 1 decimal place.



The idea

1. Use the known side-angle pair as one side of the equation.
2. Keep the unknown side above its opposite sine.

Worked Solution

$$\frac{b}{\sin 70^\circ} = \frac{9}{\sin 40^\circ}$$

$$b = \frac{9 \sin 70^\circ}{\sin 40^\circ} = 13.2 \text{ cm (1 d.p.)}$$

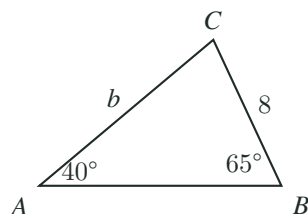
Match opposite pairs.

Multiply by $\sin 70^\circ$.

1 The sine rule

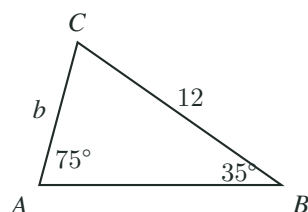
Q1 In triangle ABC , $a = 8$ cm, $A = 40^\circ$ and $B = 65^\circ$. Find b to 1 decimal place.

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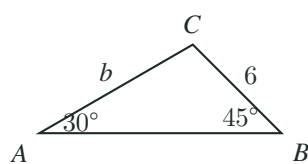
Q2 In triangle ABC , $a = 12$ cm, $A = 75^\circ$ and $B = 35^\circ$. Find b to 1 decimal place.

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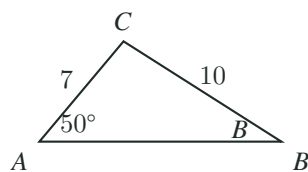


Q3 In triangle ABC , $a = 6$ cm, $A = 30^\circ$ and $B = 45^\circ$. Find b to 1 decimal place.

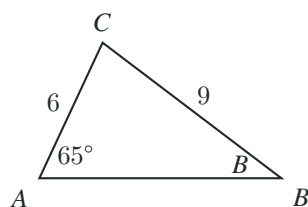
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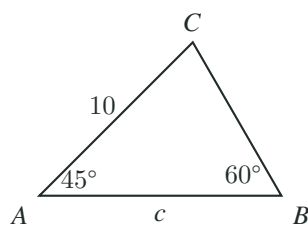
- Q4** In triangle ABC , $a = 10$ cm, $b = 7$ cm and $A = 50^\circ$. Angle B is acute. Find B to 1 decimal place. **



- Q5** In triangle ABC , $a = 9$ cm, $b = 6$ cm and $A = 65^\circ$. Angle B is acute. Find B to 1 decimal place. **



- Q6** In triangle ABC , $A = 45^\circ$, $B = 60^\circ$ and $b = 10$ cm. Find c to 1 decimal place. ***

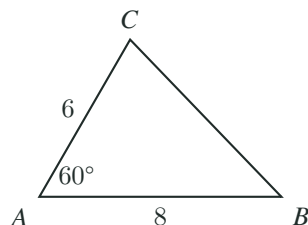


2. The cosine rule

Example Problem 2

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Two sides are 6 cm and 8 cm with included angle 60° . Find the third side to 1 decimal place.



The idea

1. The known angle lies between the two known sides.
2. Use the cosine rule to find the opposite side.

Worked Solution

$$a^2 = 6^2 + 8^2 - 2(6)(8) \cos 60^\circ = 52$$

Use the cosine rule.

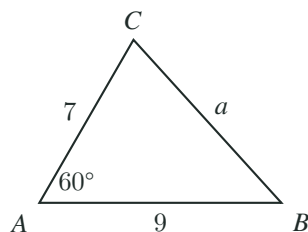
$$a = \sqrt{52} = 7.2 \text{ cm (1 d.p.)}$$

Take the positive root.

2 The cosine rule

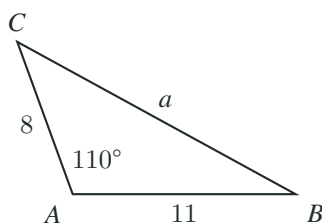
Q7 In triangle ABC , $b = 7$ cm, $c = 9$ cm and $A = 60^\circ$. Find a to 1 decimal place.

**



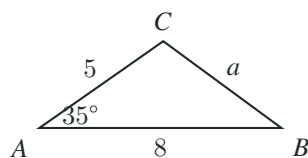
Q8 In triangle ABC , $b = 8$ cm, $c = 11$ cm and $A = 110^\circ$. Find a to 1 decimal place.

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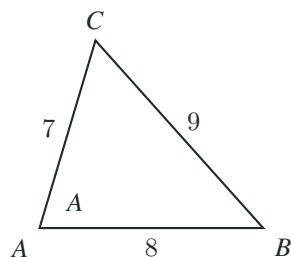


Q9 In triangle ABC , $b = 5$ cm, $c = 8$ cm and $A = 35^\circ$. Find a to 1 decimal place.

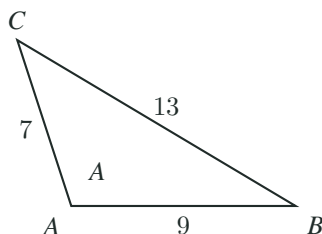
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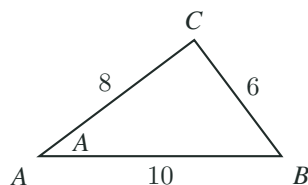
Q10 In triangle ABC , $a = 9$ cm, $b = 7$ cm and $c = 8$ cm. Find angle A to 1 decimal place.



Q11 In triangle ABC , $a = 13$ cm, $b = 7$ cm and $c = 9$ cm. Find angle A to 1 decimal place.



Q12 In triangle ABC , $a = 6$ cm, $b = 8$ cm and $c = 10$ cm. Find angle A to 1 decimal place.

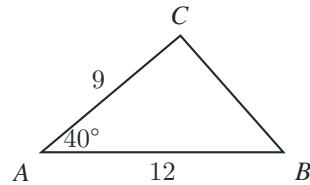


3. Triangle areas

Example Problem 3

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Find the area of a triangle with sides 9 cm and 12 cm enclosing 40° , to 1 decimal place.



The idea

1. Choose the included angle, not an angle opposite a given side.

Worked Solution

$$K = \frac{1}{2}(9)(12) \sin 40^\circ$$

Use the two sides and their included angle.

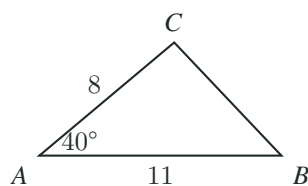
$$K = 34.7 \text{ cm}^2 \text{ (1 d.p.)}$$

Evaluate.

3 Triangle areas

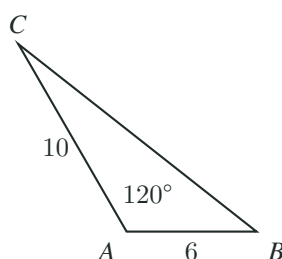
- Q13** Two sides of a triangle are 8 cm and 11 cm and their included angle is 40° . Find its area to 1 decimal place.

**



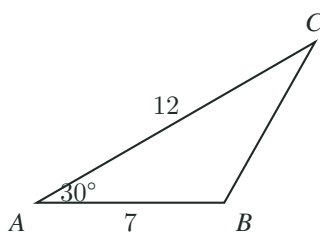
- Q14** Two sides of a triangle are 10 cm and 6 cm and their included angle is 120° . Find its area to 1 decimal place.

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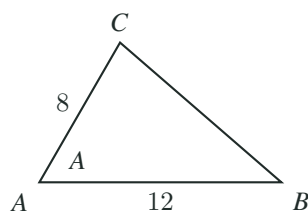


- Q15** Two sides of a triangle are 12 cm and 7 cm and their included angle is 30° . Find its area to 1 decimal place.

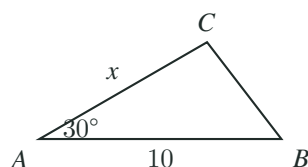
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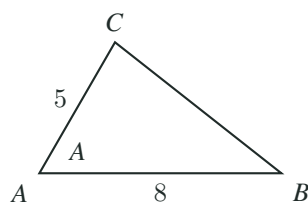
- Q16** A triangle has area 24 cm^2 . Two sides are 8 cm and 12 cm. Find both possible included angles. ***



- Q17** A triangle has area 30 cm^2 . Two sides enclosing 30° are x cm and 10 cm. Find x . ***



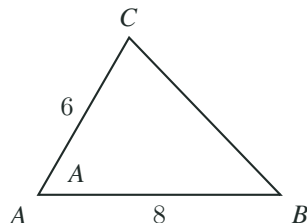
- Q18** Two sides of a triangle have lengths 5 cm and 8 cm. Find the greatest possible area and the included angle where it occurs. ***



4. Combining methods

Example Problem 4

A triangle has area 12 cm^2 and two sides 6 cm and 8 cm. Find both possible included angles.



The idea

1. Use inverse sine on the calculator to find one possible angle.
2. Also test its supplement against the triangle conditions.

Worked Solution

$$12 = 24 \sin A \Rightarrow \sin A = \frac{1}{2}$$

$$A = 30^\circ \text{ or } 150^\circ$$

$$0 < A < 180^\circ \text{ in both cases}$$

Use the area formula.

Both angles have the same sine.

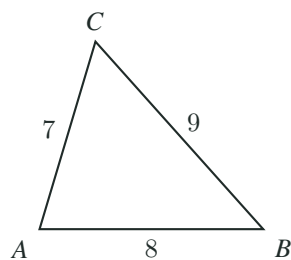
Both form a valid triangle with the two sides.

4 Combining methods

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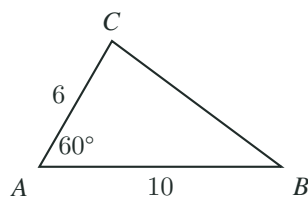
Q19 A triangle has sides 7 cm, 8 cm and 9 cm. Find its area to 1 decimal place.

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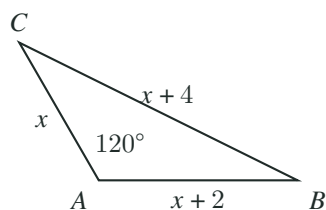


Q20 Two sides of a triangle are 6 cm and 10 cm with included angle 60° . Find its perimeter exactly.

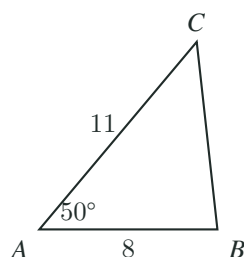
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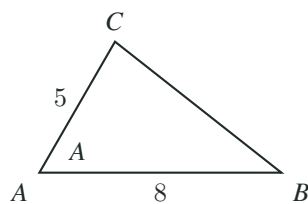
- Q21** A triangle has side lengths x , $x + 2$ and $x + 4$ cm. The angle opposite the longest side is 120° . Find x . ***



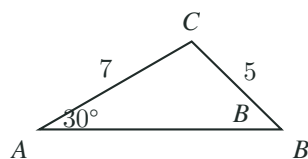
- Q22** Two paths from A to B and from A to C are 8 km and 11 km. The angle between them is 50° . A walker goes directly from B to C . How much shorter is this than travelling via A ? Give 1 decimal place. ***



- Q23** A triangle has sides 5 cm and 8 cm enclosing angle A . Its area is $10\sqrt{3} \text{ cm}^2$. Find both possible lengths of the third side.



- Q24** In triangle ABC , $a = 5 \text{ cm}$, $b = 7 \text{ cm}$ and $A = 30^\circ$. Find all possible values of B , to 1 decimal place.



Solving principles

1. Choose from the given information

A known opposite pair suggests the sine rule. Two sides and the included angle suggest the cosine rule.

2. Check the angle mode

Use degrees for GCSE triangle problems.

3. Keep exact calculator values

Carry unrounded results into a second triangle or an area calculation.

4. Check whether another triangle fits

Angles with equal sine sum to 180° . Check each possible angle against the triangle angle sum and the given information.