

GCSE MATHS

Simultaneous equations

Worked Solutions

Q1

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$$2x = 8$$

Subtract the second equation from the first.

$$x = 4$$

Divide by two.

$$y = 9 - 4 = 5$$

Substitute into the second equation.

Q2

*

$$4y = 12$$

Subtract the second equation.

$$y = 3$$

Divide by four.

$$2x = 7 + 3 = 10$$

Substitute $y = 3$.

$$x = 5$$

Divide by two.

Q3

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$$8x = 16$$

Add the equations.

$$x = 2$$

Divide by eight.

$$6 + 2y = 16$$

Substitute into the first equation.

$$y = 5$$

Subtract six, then divide by two.

Q4

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$$4x + 6y = 2$$

Multiply the first equation by two.

$$9x - 6y = 24$$

Multiply the second by three.

$$13x = 26$$

Add the equations.

$$x = 2$$

Substitute $x = 2$ into the first equation.

$$y = -1$$

Q5

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$$8x + 10y = 14$$

$$15x - 10y = 55$$

$$23x = 69$$

$$x = 3$$

$$y = -1$$

Multiply the first equation by two.

Multiply the second by five.

Add the equations.

Divide by 23, then substitute.

Q6

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$$35x - 21y = 63$$

$$6x + 21y = 60$$

$$41x = 123$$

$$x = 3$$

$$y = 2$$

Multiply the first equation by seven.

Multiply the second by three.

Add the equations.

Divide by 41, then substitute.

Q7

*

$$x + 3x - 4 = 12$$

$$4x = 16$$

$$x = 4$$

$$y = 8$$

Substitute for y .

Collect x terms and add four.

Find x , then use $y = 3x - 4$.

Q8

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$$3(2y + 1) + 4y = 23$$

$$10y + 3 = 23$$

$$y = 2$$

$$x = 5$$

Substitute for x .

Expand and collect terms.

Subtract three, divide by ten, then substitute.

Q9

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$$x = 3y$$

$$\frac{3y}{2} + y = 5$$

$$5y = 10$$

$$y = 2$$

$$x = 6$$

Isolate x in the second equation.

Substitute into the first equation.

Multiply every term by two.

Find y and then x .

Q10

**

$$4x + 3y = 23$$

$$x = y + 4$$

$$7y + 16 = 23$$

$$y = 1$$

$$x = 5$$

Multiply the first equation by ten.

Rearrange the second.

Substitute and expand.

Subtract 16, divide by seven, then substitute.

Q11

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$$x + y = \frac{7}{2}$$

$$4x = 4$$

$$x = 1$$

$$y = \frac{5}{2}$$

Divide the first equation by two.

Add this to the second equation.

Substitute into $x + y = 7/2$.

Q12

Extension — beyond GCSE

$$3(2x + ky) = 24$$

$$3k = 9$$

$$k = 3$$

Match the coefficient of x by multiplying by three.

The y coefficients must also match.

Both equations then describe the same line.

Q13

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$$2a + 3c = 31$$

$$4a + c = 37$$

$$4a + 6c = 62$$

$$5c = 25$$

$$c = 5$$

$$a = 8$$

Let a and c be the prices in pounds.

Double the first equation.

Subtract the second equation.

Find c , then use $4a + c = 37$.

Q14

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$$l + w = 23$$

$$l - w = 5$$

$$2l = 28$$

$$l = 14$$

$$w = 9$$

$$A = 14 \times 9 = 126$$

Halve the perimeter; write the difference.

Add the equations.

Find the dimensions.

Multiply length by width.

Q15

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$$a + b = 26$$

$$20a + 50b = 820$$

$$20a + 20b = 520$$

$$30b = 300$$

$$b = 10$$

Let a and b count the 20p and 50p coins.

Multiply the first equation by 20.

Subtract to eliminate a .

Divide by 30.

Q16

$$f + 6r = 11.4$$

$$f + 14r = 21.8$$

$$8r = 10.4$$

$$r = 1.3$$

$$f = 11.4 - 7.8 = 3.6$$

$$3.6 + 19(1.3) = 28.3$$

Let f be the fee and r the price per kilometre.

Subtract, then divide by eight.

Use the 6 km journey.

Add the fee and the charge for 19 km.

Q17

$$a + b = 11$$

$$(10b + a) - (10a + b) = 27$$

$$b - a = 3$$

$$2b = 14$$

$$b = 7$$

$$a = 4$$

$$10a + b = 47$$

Let a be the tens digit and b the units digit.

Write the increase after reversal.

Simplify and divide by nine.

Add the two equations.

Find the original number.

Q18

$$p - 5 = 4(c - 5)$$

$$p + 7 = 2(c + 7)$$

$$p = 4c - 15$$

$$p = 2c + 7$$

$$2c = 22$$

$$c = 11$$

$$p = 29$$

Write the condition five years ago.

Write the condition in seven years.

Expand and isolate p .

Equate the expressions for p .

Substitute $c = 11$.

Q19

**

$$x^2 = x + 6$$

$$(x - 3)(x + 2) = 0$$

$$x = -2 \Rightarrow y = 4$$

$$x = 3 \Rightarrow y = 9$$

Equate the two expressions for y .

Rearrange and factorise.

Substitute the first root.

Substitute the second root.

Q20

$$y = 7 - x$$

$$x(7 - x) = 10$$

$$(x - 2)(x - 5) = 0$$

$$(x, y) = (2, 5) \text{ or } (5, 2)$$

Isolate y .

Substitute into $xy=10$.

Rearrange and factorise.

Match each root with $y = 7 - x$.

Q21

$$x^2 + (2x + 1)^2 = 25$$

$$5x^2 + 4x - 24 = 0$$

$$x = \frac{-4 \pm \sqrt{496}}{10}$$

$$x = \frac{-2 \pm 2\sqrt{31}}{5}$$

$$y = \frac{1 \pm 4\sqrt{31}}{5}$$

Substitute the linear equation.

Expand and collect terms.

Use the quadratic formula.

Simplify the square root.

Use the matching sign in $y = 2x + 1$.

Q22

$$x^2 + (x + 1)^2 = 13$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$(x, y) = (2, 3) \text{ or } (-3, -2)$$

Substitute for y .

Expand, subtract 13 and divide by two.

Factorise.

Use $y = x + 1$ for each root.

Q23

$$x^2 - 2x + 3 - c = 0$$

$$(x - 1)^2 = c - 2$$

$$c - 2 = 0$$

$$c = 2$$

$$(x, y) = (1, 4)$$

Equate the two expressions for y .

Complete the square.

Exactly one root requires the square to equal zero.

Find $x = 1$ and substitute.

Q24

$$xy = 60$$

$$x^2 + y^2 = 169$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$(x + y)^2 = 169 + 120 = 289$$

$$x + y = 17$$

$$P = 34 \text{ cm}$$

Use area and Pythagoras for positive side lengths.

Expand the square of the sum.

Substitute both conditions.

Take the positive root, then double.