

GCSE MATHS

Simultaneous equations

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

One pair, two conditions

A solution must satisfy both equations. For $x + y = 9$ and $x - y = 3$, the pair is $x = 6, y = 3$.

Elimination

Multiply whole equations to make one pair of coefficients equal. Subtract when the signs match; add when they are opposite.

Substitution

If $y = 2x - 1$, replace every y in the other equation by $(2x - 1)$.

Quadratic intersections

Substitution may give a quadratic. Solve it, then find the matching second coordinate for each root.

1. Eliminating a variable

Example Problem 1

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Solve $2x + 3y = 18$ and $3x - 2y = 1$.

The idea

1. Multiply the first equation by two and the second by three to obtain opposite y coefficients.
2. Add, find x , then substitute into an original equation.

Worked Solution

$$4x + 6y = 36$$

Multiply the first equation by two.

$$9x - 6y = 3$$

Multiply the second by three.

$$13x = 39$$

Add to eliminate y .

$$x = 3$$

$$6 + 3y = 18$$

Substitute $x = 3$ into the first equation.

$$y = 4$$

1 Eliminating a variable

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Q1 Solve $3x + y = 17$ and $x + y = 9$.

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Q2 Solve $2x + 3y = 19$ and $2x - y = 7$.

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Q3 Solve $3x + 2y = 16$ and $5x - 2y = 0$.

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Q4 Solve $2x + 3y = 1$ and $3x - 2y = 8$.

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Q5 Solve $4x + 5y = 7$ and $3x - 2y = 11$.

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Q6 Solve $5x - 3y = 9$ and $2x + 7y = 20$.

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2. Substitution and fractions

Example Problem 2

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Solve $y = 2x - 3$ and $\frac{x}{2} + y = 7$.

The idea

1. Replace y in the second equation by $2x - 3$.
2. Multiply the resulting equation by two to remove the fraction.

Worked Solution

$$\frac{x}{2} + 2x - 3 = 7$$

Replace y by $2x - 3$.

$$x + 4x - 6 = 14$$

Multiply every term by two.

$$5x = 20$$

Collect terms and solve.

$$x = 4$$

$$y = 2(4) - 3 = 5$$

Use the first equation.

2. Substitution and fractions

Example Problem 3

Extension — beyond GCSE

The equations $3x + 2y = 12$ and $6x + py = 24$ describe the same line. Find p and explain why there are infinitely many solutions.

The idea

1. If one whole equation is a non-zero multiple of the other, both impose the same condition.
2. Every point on that line then satisfies both equations.

Worked Solution

$$2(3x + 2y) = 24$$

Multiply the first equation by two.

$$6x + 4y = 24$$

Match the second equation.

$$p = 4$$

$$y = 6 - \frac{3}{2}x$$

Every real choice of x gives a matching y ; there are infinitely many pairs.

2 Substitution and fractions

Q7 Solve $y = 3x - 4$ and $x + y = 12$.

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Q8 Solve $x = 2y + 1$ and $3x + 4y = 23$.

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Q9 Solve $\frac{x}{2} + y = 5$ and $x - 3y = 0$.

**

Q10 Solve $0.4x + 0.3y = 2.3$ and $x - y = 4$.

**

Q11 Solve $2(x + y) = 7$ and $3x - y = \frac{1}{2}$.

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Q12 Extension — beyond GCSE

The equations $2x + ky = 8$ and $6x + 9y = 24$ have infinitely many solutions. Find k and explain why.

3. Forming two equations

Example Problem 4

A cafe sells 18 drinks: teas at £2 and coffees at £3. The total is £47. Find how many of each were sold.

The idea

1. Use one equation for the number of drinks and one for their cost.
2. Double the first equation so subtraction eliminates t .

Worked Solution

$$t + c = 18$$

Count the drinks.

$$2t + 3c = 47$$

Write the cost in pounds.

$$2t + 2c = 36$$

Double the number equation.

$$c = 11$$

Subtract, then use $t + c = 18$.

$$t = 7$$

3 Forming two equations

Q13 Two adult tickets and three child tickets cost £31. Four adult tickets and one child ticket cost £37. Find each ticket price.

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Q14 A rectangle has perimeter 46 cm. Its length is 5 cm more than its width. Find its area.

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Q15 A jar holds 26 coins, all 20p or 50p. Their total value is £8.20. How many 50p coins are there?

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Q16 A taxi charges a fixed fee plus a cost per kilometre. A 6 km journey costs £11.40 and a 14 km journey costs £21.80. Find the cost of a 19 km journey.

Q17 A two-digit number has digits that sum to 11. Reversing the digits increases the number by 27. Find the original number.

Q18 Five years ago, a parent was four times their child's age. In seven years, the parent will be twice the child's age. Find their present ages.

4. Linear and quadratic intersections

Example Problem 5

Find the intersection points of $y = x + 2$ and $y = x^2 - 4$.

The idea

1. Equate the expressions for y to obtain a quadratic in x .
2. Solve both factors, then find both y values.

Worked Solution

$$x^2 - 4 = x + 2$$

Both expressions equal y .

$$x^2 - x - 6 = 0$$

Collect all terms on one side.

$$(x - 3)(x + 2) = 0$$

Factorise.

$$x = 3 \Rightarrow y = 5$$

Use the line to find y .

$$x = -2 \Rightarrow y = 0$$

Keep the second pair together.

4 Linear and quadratic intersections

★★★★

Q19 Find the intersection points of $y = x^2$ and $y = x + 6$.

★★

Q20 Solve $x + y = 7$ and $xy = 10$.

★★★

Q21 Find the intersection points of $y = 2x + 1$ and $x^2 + y^2 = 25$.

★★★

Q22 Solve $y = x + 1$ and $x^2 + y^2 = 13$.

Q23 The line $y = 2x + c$ touches the curve $y = x^2 + 3$ at exactly one point. Find c and the point of contact.

Q24 A rectangle has area 60 cm^2 and diagonal 13 cm . Find its perimeter.

Solving principles

1. Keep both equations true

Multiplying an equation means multiplying every term, including its right-hand side.

2. Choose a method

If one variable is already the subject, try substitution. If coefficients are easy to match, try elimination.

3. Keep pairs together

Write each solution as a pair. Do not mix the x from one solution with the y from another.

4. Check the context

Substitute into both original equations. Reject negative lengths or fractional numbers of tickets.