

GCSE MATHS

Similarity

Worked Solutions

Q1

★

$$k = \frac{15}{6} = \frac{5}{2}$$

$$EF = 8 \times \frac{5}{2} = 20 \text{ cm}$$

Scale from ABC to DEF . BC corresponds to EF .**Q2**

★★

$$k = \frac{9}{15} = \frac{3}{5}$$

$$w = 10 \times \frac{3}{5} = 6 \text{ cm}$$

Scale from larger to smaller.

Apply the factor to the corresponding width.

Q3

★★

$$k = \frac{7}{3}$$

$$P = 24 \times \frac{7}{3} = 56 \text{ cm}$$

Every side uses the same length factor.

Perimeter scales like length.

Q4

★★

$$L = 11.5(40) = 460 \text{ cm}$$

$$L = 4.6 \text{ m}$$

Multiply by the scale factor.

Divide centimetres by 100.

Q5

★★

$$k = \frac{14}{2} = 7$$

$$h = 1.5(7) = 10.5 \text{ m}$$

The equal sunlight angle gives similar right triangles.

Apply the shadow scale factor to the height.

Q6

$$\frac{3x-1}{x+2} = \frac{100}{40} = \frac{5}{2}$$

$$2(3x-1) = 5(x+2)$$

$$x = 12$$

Corresponding sides have the perimeter ratio.

Cross-multiply.

Expand and solve.

Q7

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$$k^2 = 3^2 = 9$$

$$A = 12(9) = 108 \text{ cm}^2$$

Square the length factor.

Multiply the smaller area by nine.

Q8

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$$\frac{A_s}{A_l} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$A_s = 150 \times \frac{4}{25} = 24 \text{ cm}^2$$

Square the smaller-to-larger length ratio.

Scale down the area.

Q9

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$$k = \sqrt{\frac{81}{36}} = \frac{3}{2}$$

$$L = 8 \times \frac{3}{2} = 12 \text{ cm}$$

Take the square root of the area ratio.

Apply the length factor.

Q10

$$\frac{L_l}{L_s} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

$$P_l = 27 \times \frac{5}{3} = 45 \text{ cm}$$

Convert area ratio to length ratio.

Perimeter uses the length factor.

Q11

$$k = 1.2$$

$$k^2 = 1.44$$

$$1.44 - 1 = 0.44 = 44\%$$

Square the enlargement factor.

Compare the new area with the original.

Q12

$$k = \sqrt{\frac{48}{108}} = \frac{2}{3}$$

$$P_s = 42 \times \frac{2}{3} = 28 \text{ cm}$$

Scale from larger to smaller.

Use the length scale factor.

Q13

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$$k^3 = 2^3 = 8$$

$$V = 35(8) = 280 \text{ cm}^3$$

Cube the length factor.

Apply the volume factor.

Q14

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$$\frac{V_s}{V_l} = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$V_s = 162 \times \frac{8}{27} = 48 \text{ cm}^3$$

Cube the length ratio.

Multiply by the volume fraction.

Q15

$$k = \sqrt[3]{\frac{216}{64}} = \frac{3}{2}$$

$$h_l = 10 \times \frac{3}{2} = 15 \text{ cm}$$

Take the cube root of the volume ratio.

Apply the length factor.

Q16

$$k = \sqrt[3]{\frac{125}{8}} = \frac{5}{2}$$

$$k^2 = \frac{25}{4}$$

$$A_l = 48 \times \frac{25}{4} = 300 \text{ cm}^2$$

Convert volume ratio to length ratio.

Square to obtain the surface-area factor.

Apply that factor to the area.

Q17

$$k^3 = 1.1^3 = 1.331$$

$$1.331 - 1 = 0.331 = 33.1\%$$

Cube the length multiplier.

Find the increase relative to the original volume.

Q18

$$\frac{V_l}{V_s} = \frac{2000}{250} = 8$$

Equal density makes mass proportional to volume.

$$k = \sqrt[3]{8} = 2$$

Take the cube root.

$$h_l = 7(2) = 14 \text{ cm}$$

Scale the height.

Q19

$$AB = 6 + 3 = 9 \text{ cm}$$

Use the whole corresponding side.

$$\frac{DE}{12} = \frac{6}{9} = \frac{2}{3}$$

Triangles ADE and ABC are similar.

$$DE = 8 \text{ cm}$$

Multiply by 12.

Q20

$$\frac{AD}{AB} = \frac{2}{5}$$

The whole side has five parts.

$$A_{ADE} = 75 \left(\frac{2}{5} \right)^2 = 12$$

Square the length fraction for the smaller triangle.

$$A_{DBCE} = 75 - 12 = 63 \text{ cm}^2$$

Subtract the smaller triangle.

Q21

$$h_s : h_l = 2 : 3$$

Take the square roots of the area ratio.

$$3t - 2t = 7$$

The difference is one ratio part.

$$t = 7$$

$$h_s = 14$$

Find two parts and three parts.

$$h_l = 21$$

Q22

$$\frac{V_s}{V_l} = \left(\frac{1}{3} \right)^3 = \frac{1}{27}$$

The cones are similar.

$$V_s = 216 \div 27 = 8 \text{ cm}^3$$

Find the removed cone volume.

$$V_{\text{remaining}} = 216 - 8 = 208 \text{ cm}^3$$

Subtract from the original.

Q23

$$k = \frac{2}{6} = \frac{1}{3}$$

$$h_s = 5 \text{ cm}$$

$$V_l = \frac{1}{3}\pi(6^2)(15) = 180\pi$$

$$V_s = \frac{1}{3}\pi(2^2)(5) = \frac{20\pi}{3}$$

$$V_f = 180\pi - \frac{20\pi}{3} = \frac{520\pi}{3}$$

Use the radius ratio to find the removed cone height.

Find the original cone volume.

Find the removed cone volume.

Subtract the volumes.

Q24

$$L_s : L_l = 3 : 4$$

$$V_s : V_l = 27 : 64$$

$$37t = 74$$

$$t = 2$$

$$V_s = 54$$

$$V_l = 128 \text{ litres}$$

Take square roots of the area ratio.

Cube the length ratio.

The capacity difference is 37 ratio parts.

Multiply each volume ratio part by two.