

Similarity

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Correspondence

Similar shapes have equal corresponding angles and proportional corresponding lengths. Match vertices before using lengths.

Length and area

For length scale factor k , the area scale factor is k^2 .

Length and volume

For similar solids with length scale factor k , the volume scale factor is k^3 .

Parallel lines in triangles

A line parallel to one side of a triangle forms a smaller triangle with equal corresponding angles. Compare whole corresponding sides.

A calculator may be used. Diagrams are not drawn to scale.

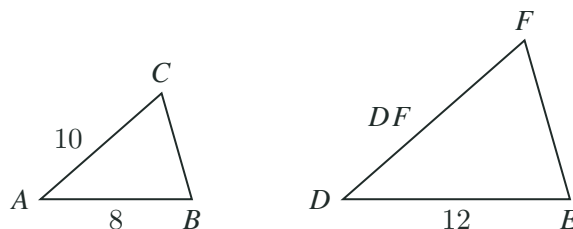
Use the given measurements to calculate answers.

1. Corresponding lengths

Example Problem 1

**

Triangles ABC and DEF are similar: A corresponds to D , B to E and C to F . $AB = 8$ cm, $DE = 12$ cm and $AC = 10$ cm. Find DF .



The idea

1. Match vertices in the order given.
2. Use one corresponding pair to scale the other pair.

Worked Solution

$$k = \frac{12}{8} = \frac{3}{2}$$

$$DF = 10 \times \frac{3}{2} = 15 \text{ cm}$$

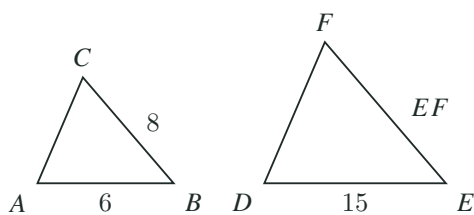
AB corresponds to DE .

AC corresponds to DF .

1 Corresponding lengths

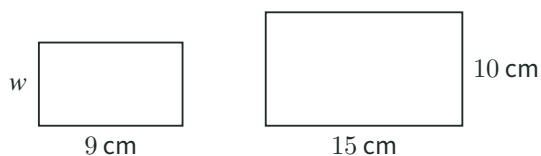
- Q1** Triangles ABC and DEF are similar, with A corresponding to D , B to E and C to F . $AB = 6$ cm, $DE = 15$ cm and $BC = 8$ cm. Find EF .

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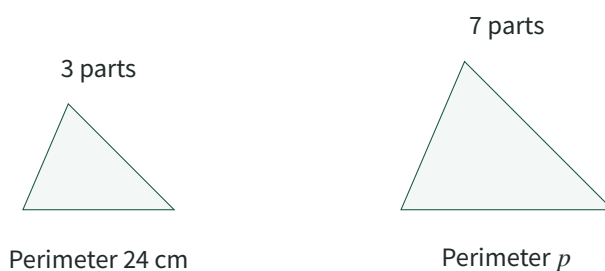


- Q2** Two similar rectangles have corresponding lengths 9 cm and 15 cm. The larger rectangle has width 10 cm. Find the smaller width.

**



- Q3** Two similar triangles have side lengths in the ratio 3 : 7. The smaller has perimeter 24 cm. Find the larger perimeter. **



- Q4** A model car is built to a scale of 1 : 40. Its length is 11.5 cm. Find the real car's length in metres. **

- Q5** A 1.5 m pole casts a 2 m shadow. At the same time a nearby vertical tree on the same horizontal ground casts a 14 m shadow. Find the tree height. **

- Q6** Two similar triangles have corresponding sides $(x + 2)$ cm and $(3x - 1)$ cm. Their perimeters are 40 cm and 100 cm respectively. Find x . ***



2. Area scale factors

Example Problem 2

Two similar shapes have areas 50 cm^2 and 98 cm^2 . A side of the smaller is 15 cm. Find the corresponding larger side.



The idea

1. Take a square root to convert area scaling into length scaling.
2. Multiply the known side by that length factor.

Worked Solution

$$k^2 = \frac{98}{50} = \frac{49}{25}$$

$$k = \frac{7}{5}$$

$$L = 15 \times \frac{7}{5} = 21 \text{ cm}$$

Find the area factor.

Take its positive square root.

Use the length factor.

2 Area scale factors

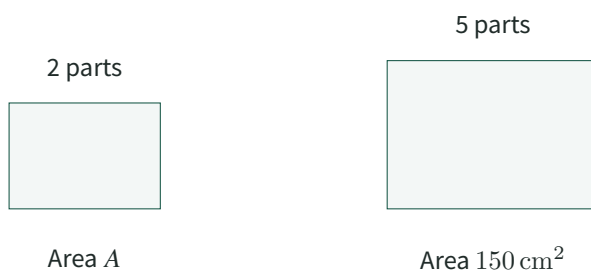
- Q7** Two similar shapes have length scale factor 3 from smaller to larger. The smaller area is 12 cm^2 . Find the larger area.

**



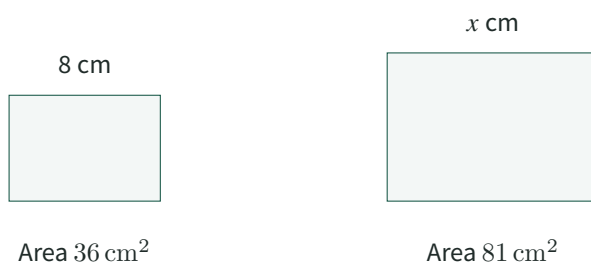
- Q8** Two similar shapes have corresponding lengths in the ratio 2 : 5. The larger area is 150 cm^2 . Find the smaller area.

**

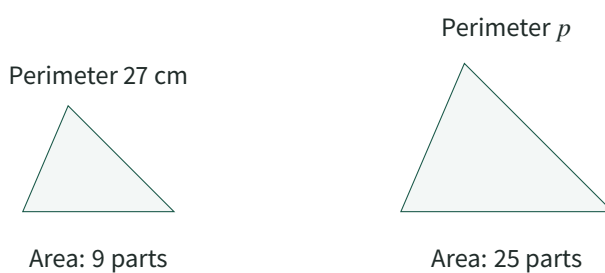


- Q9** Two similar shapes have areas 36 cm^2 and 81 cm^2 . A side of the smaller is 8 cm. Find the corresponding side of the larger.

**



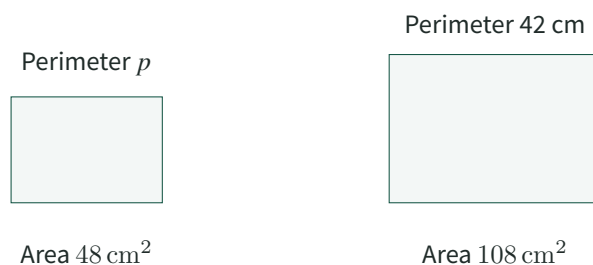
- Q10** Two similar triangles have areas in the ratio 9 : 25. The smaller perimeter is 27 cm. Find the larger perimeter.



- Q11** Every length of a shape is increased by 20%. Find the percentage increase in its area.



- Q12** Two similar rectangles have areas 48 cm^2 and 108 cm^2 . The larger perimeter is 42 cm. Find the smaller perimeter.



3. Volume scale factors

Example Problem 3

Two similar solids have volumes 81 cm^3 and 192 cm^3 . The smaller surface area is 45 cm^2 . Find the larger surface area.

The idea

1. Convert volume to length with a cube root.
2. Square the length factor before applying it to an area.

Worked Solution

$$k^3 = \frac{192}{81} = \frac{64}{27}$$

$$k = \frac{4}{3}$$

$$A_l = 45 \left(\frac{4}{3}\right)^2 = 80 \text{ cm}^2$$

Simplify the volume ratio.

Take the cube root.

Square the length factor to scale surface area.

3 Volume scale factors

Q13 Two similar solids have length scale factor 2 from smaller to larger. The smaller volume is 35 cm^3 . Find the larger volume.

**

Q14 Two similar solids have corresponding lengths in the ratio 2 : 3. The larger volume is 162 cm^3 . Find the smaller volume.

**

Q15 Two similar solids have volumes 64 cm^3 and 216 cm^3 . A height of the smaller is 10 cm. Find the corresponding height of the larger.

- Q16** Two similar solids have volumes in the ratio 8 : 125. The smaller surface area is 48 cm^2 . Find the larger surface area. ***

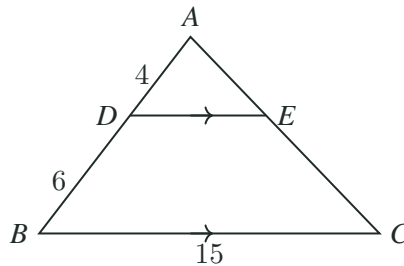
- Q17** A solid is enlarged so that every length rises by 10%. Find the percentage increase in volume. ***

- Q18** Two similar solid ornaments are made of the same material with uniform density. Their masses are 250 g and 2 kg. The smaller has height 7 cm. Find the larger height. ***

4. Connected shapes and reasoning

Example Problem 4

In triangle ABC , D lies on AB and E lies on AC , with DE parallel to BC . $AD = 4$ cm, $DB = 6$ cm and $BC = 15$ cm. Find DE .



The idea

1. Compare the small triangle with the entire large triangle.
2. Use AD divided by AB , not AD divided by DB .

Worked Solution

$$AB = 4 + 6 = 10 \text{ cm}$$

$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{10}$$

$$DE = 15 \times \frac{4}{10} = 6 \text{ cm}$$

Add both segments to find the whole side.

Parallel lines give equal corresponding angles and similar triangles.

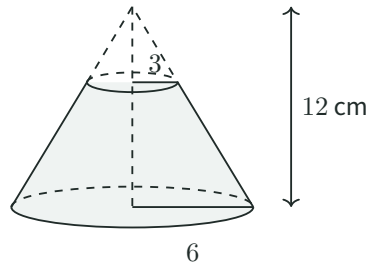
Apply the smaller-to-larger length ratio.

4. Connected shapes and reasoning

Example Problem 5

★★★★

A cone has height 12 cm and base radius 6 cm. A parallel cut leaves a frustum with top radius 3 cm. Find its exact volume.



The idea

1. Use the radius ratio to find the height of the removed cone.
2. Subtract the smaller cone volume from the original volume.

Worked Solution

$$k = \frac{3}{6} = \frac{1}{2}$$

The cut produces similar cones.

$$h_s = 6 \text{ cm}$$

$$V_l = \frac{1}{3}\pi(6^2)(12) = 144\pi$$

Find the original cone volume.

$$V_s = \frac{1}{3}\pi(3^2)(6) = 18\pi$$

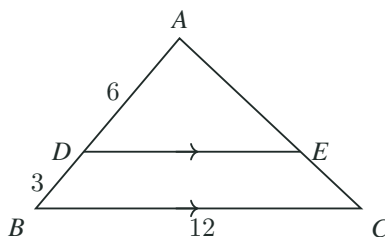
Find the removed cone volume.

$$V_f = 144\pi - 18\pi = 126\pi \text{ cm}^3$$

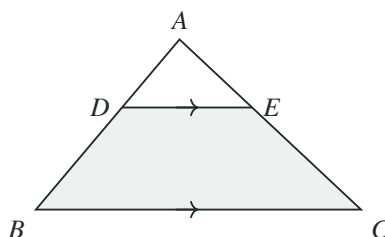
Subtract to leave the frustum.

4 Connected shapes and reasoning

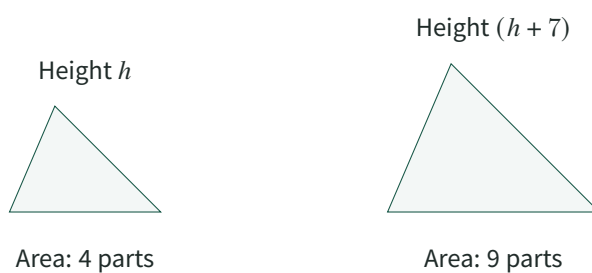
- Q19** In triangle ABC , D lies on AB and E lies on AC , with DE parallel to BC . $AD = 6$ cm, $DB = 3$ cm and $BC = 12$ cm. Find DE .



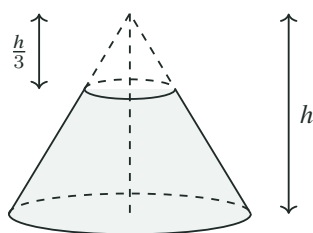
- Q20** In triangle ABC , D lies on AB and E lies on AC , with DE parallel to BC . $AD : DB = 2 : 3$. The area of triangle ABC is 75 cm^2 . Find the area of trapezium $DBCE$.



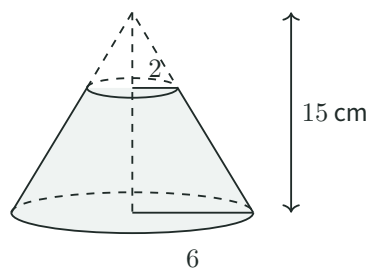
- Q21** Two similar triangles have areas in the ratio 4 : 9. Their corresponding heights differ by 7 cm. Find both heights.



- Q22** A cone is cut parallel to its base, removing a smaller cone from the top. The removed cone's height is one third of the original height. The original volume is 216 cm^3 . Find the remaining volume.



- Q23** A cone is cut parallel to its base to form a frustum. The original cone has height 15 cm and base radius 6 cm. The frustum's top radius is 2 cm. Find the exact volume of the frustum. ★ ★ ★ ★



- Q24** Two similar solid containers have surface areas in the ratio 9 : 16. Their capacities differ by 74 litres. Find both capacities. ★ ★ ★ ★

Solving principles

1. State the direction

Write whether the scale factor takes the smaller shape to the larger or the reverse.

2. Use the right power

Lengths use k , areas use k^2 , and volumes use k^3 .

3. Include the whole side

A marked segment may be only part of a corresponding side; add or subtract before forming the ratio.

4. Separate a removed shape

For a frustum or a region between similar shapes, subtract areas or volumes after applying the scale factor.