

GCSE MATHS

Sequences

Worked Solutions

Q1

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$$d = 4$$

The consecutive difference is four.

$$u_n = 4n + c$$

Use the first term.

$$7 = 4 + c$$

$$u_n = 4n + 3$$

The constant is three.

Q2

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$$d = -5$$

The sequence decreases by five.

$$u_n = -5n + c$$

Use $n = 1$.

$$18 = -5 + c$$

$$u_n = 23 - 5n$$

The constant is 23.

Q3

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$$u_{40} = 6(40) - 1$$

Substitute $n = 40$.

$$u_{40} = 239$$

Multiply, then subtract one.

Q4

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$$9n + 4 = 256$$

Set the formula equal to the proposed term.

$$9n = 252$$

Solve for the position.

$$n = 28$$

$$u_{28} = 256$$

The position is a positive integer.

Q5

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$$5d = 39 - 14 = 25$$

There are five steps between positions three and eight.

$$d = 5$$

Divide by five.

$$14 = 5(3) + c$$

Use the third term.

$$c = -1$$

$$u_n = 5n - 1$$

Write the rule.

Q6

$$83 - 4n < 0$$

A negative term is less than zero.

$$n > \frac{83}{4} = 20.75$$

Solve the inequality.

$$n = 21$$

Use the first whole number above 20.75.

Q7

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$$4 = 2^2$$

Recognise consecutive squares.

$$9 = 3^2$$

$$16 = 4^2$$

$$u_n = (n + 1)^2$$

The square base is one more than the position.

Q8

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$$\text{first differences} = 5, 7, 9$$

Find first and second differences.

$$\text{second differences} = 2$$

$$u_n - n^2 = 2, 4, 6, 8, \dots$$

The quadratic coefficient is one.

$$u_n = n^2 + 2n$$

Add the linear remainder back.

Q9

first differences = 9, 15, 21

second differences = 6

$$a = \frac{6}{2} = 3$$

$$u_n - 3n^2 = 2, 2, 2, 2, \dots$$

$$u_n = 3n^2 + 2$$

Calculate the differences.

Halve the second difference.

Subtract $3n^2$.

Add the constant remainder.

Q10

first differences = 5, 9, 13

second differences = 4

$$u_n - 2n^2 = 0, -1, -2, -3, \dots$$

$$0, -1, -2, -3 : \quad 1 - n$$

$$u_n = 2n^2 - n + 1$$

Find the second difference.

Subtract the quadratic part.

Find the linear remainder.

Combine the parts.

Q11

$$n^2 + 3n - 154 = 0$$

$$(n + 14)(n - 11) = 0$$

$$n = 11$$

Set the rule equal to 154.

Factorise.

Reject the negative position.

Q12

second differences = 4

$$a = 2$$

$$u_n - 2n^2 = 4, 7, 10, 13, \dots$$

$$u_n = 2n^2 + 3n + 1$$

$$u_{20} = 800 + 60 + 1 = 861$$

First differences are 9, 13 and 17.

Find the remainder after subtracting $2n^2$.The remainder has rule $3n + 1$.Substitute $n = 20$.

Q13

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$$r = 2$$

Each term is twice the previous one.

$$24(2) = 48$$

Apply the multiplier twice.

$$48(2) = 96$$

Q14

★★

$$u_1 = 5$$

Identify the first term and constant multiplier.

$$r = 3$$

$$u_n = 5 \times 3^{n-1}$$

There are $n - 1$ multiplications from the first term.

Q15

★★

$$u_n = 64 \left(\frac{1}{2} \right)^{n-1}$$

The multiplier is one half.

$$u_6 = 64 \left(\frac{1}{2} \right)^5 = 2$$

Use five halvings, not six.

Q16

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$$u_2 = 2(4) - 1 = 7$$

Apply the rule to the first term.

$$u_3 = 2(7) - 1 = 13$$

Use the updated term.

$$u_4 = 2(13) - 1 = 25$$

Apply the rule again.

Q17

★★★

$$u_3 = 5 + 2 = 7$$

Add the preceding two terms.

$$u_4 = 7 + 5 = 12$$

Move one position along.

$$u_5 = 12 + 7 = 19$$

Apply the recurrence again.

$$u_6 = 19 + 12 = 31$$

Add the two most recent terms.

Q18

$$r^3 = \frac{324}{12} = 27$$

There are three multiplications between the given terms.

$$r = 3$$

Take the cube root.

$$u_1 = 12 \div 3 = 4$$

Undo one multiplication from the second term.

Q19

$$n^2 + n - 90 = 0$$

Set the formula equal to 91.

$$(n + 10)(n - 9) = 0$$

Factorise.

$$n = 9$$

This is a positive whole-number position.

Q20

$$n^2 + 2 = 100 \Rightarrow n^2 = 98$$

Solve for the square of the position.

$$9^2 < 98 < 10^2$$

Locate 98 between consecutive squares.

No integer value of n

There is no whole-number position.

Q21

$$4n + 7 = 7n - 5$$

Use the same position in both rules.

$$12 = 3n$$

Solve the linear equation.

$$n = 4$$

$$u_4 = 23$$

Substitute to find the common value.

Q22

$$n^2 + 2n = 8n - 5$$

Equate the terms at the same position.

$$n^2 - 6n + 5 = 0$$

Collect terms on one side.

$$(n - 1)(n - 5) = 0$$

Factorise.

$$n = 1 \text{ or } 5$$

Both are valid positions.

Q23

$$u_n = n(n+1)$$

n and $n+1$ are consecutive

$n(n+1)$ is even

Factorise the n th term.

Of two consecutive integers, one is odd and the other is even.

One factor is even, so the product is even.

Q24

(a) $u_{n+1} - u_n = 3(n+1)^2 + (n+1) - (3n^2 + n)$

$$u_{n+1} - u_n = 6n + 4$$

$$6n + 4 = 100$$

$$n = 16$$

(b) $u_{16} = 784$

$$u_{17} = 884$$

Write the next term using $n+1$.

Expand and simplify.

Find the earlier position.

Substitute both consecutive positions.