

Sequences

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Arithmetic sequences

u_n is the term at position n . A constant difference gives $u_n = dn + c$. Find d from consecutive terms, then use a known term to find c .

Quadratic sequences

For $u_n = an^2 + bn + c$, the second difference is $2a$. For example, n^2 gives 1, 4, 9, 16: its first differences are 3, 5, 7 and its second difference is 2. Subtract an^2 from the terms to find the remaining linear rule.

Geometric sequences

A constant multiplier r gives $u_n = u_1 r^{n-1}$. The first term has index $n = 1$.

Recursive rules

$u_{n+1} = 2u_n + 3$ gives the next term from the current term. A starting value is needed.

1. Arithmetic sequences

Example Problem 1

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An arithmetic sequence has second term 9 and sixth term 29. Find its n th term.

The idea

1. Divide the change in value by the change in position.
2. Use a known position to find the constant term.

Worked Solution

$$4d = 29 - 9 = 20$$

There are four equal steps.

$$d = 5$$

Find one step.

$$9 = 5(2) + c$$

Use the second term.

$$c = -1$$

$$u_n = 5n - 1$$

Write the full rule.

1 Arithmetic sequences

Q1 Find the n th term of 7, 11, 15, 19, ...

*

Q2 Find the n th term of 18, 13, 8, 3, ...

*

Q3 The n th term is $6n - 1$. Find the 40th term.

**

Q4 The n th term is $9n + 4$. Is 256 a term? Show your reasoning.

**

Q5 An arithmetic sequence has third term 14 and eighth term 39. Find its n th term.

**

Q6 The n th term is $83 - 4n$. Find the position of the first negative term.

2. Quadratic sequences

Example Problem 2

Find the n th term of 4, 11, 22, 37, ...

The idea

1. Use half the second difference as the coefficient of n^2 .
2. Subtract those square terms, then find the linear remainder.

Worked Solution

first differences = 7, 11, 15

second differences = 4

$$a = 2$$

$$u_n - 2n^2 = 2, 3, 4, 5, \dots$$

$$u_n = 2n^2 + n + 1$$

Calculate consecutive differences.

Halve the second difference.

Subtract the quadratic contribution.

The remainder has rule $n + 1$.

2 Quadratic sequences

Q7 Find the n th term of 4, 9, 16, 25, ...

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Q8 Find the n th term of 3, 8, 15, 24, ...

**

Q9 Find the n th term of 5, 14, 29, 50, ...

Q10 Find the n th term of 2, 7, 16, 29, ...

Q11 The n th term is $n^2 + 3n$. Find the position of the term 154.

Q12 A quadratic sequence begins 6, 15, 28, 45, ... Find its 20th term.

3. Geometric and recursive sequences

Example Problem 3

A sequence has $u_1 = 3$ and $u_{n+1} = 3u_n + 2$. Find u_4 .

The idea

1. Calculate each term in turn.
2. Replace the previous input by the result each time.

Worked Solution

$$u_2 = 3(3) + 2 = 11$$

Use the first term as the input.

$$u_3 = 3(11) + 2 = 35$$

Use the new term, 11.

$$u_4 = 3(35) + 2 = 107$$

Repeat the same operation.

3 Geometric and recursive sequences

Q13 Find the next two terms of 3, 6, 12, 24, ...

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Q14 Find the n th term of 5, 15, 45, 135, ...

**

Q15 Find the sixth term of 64, 32, 16, 8, ...

**

Q16 A sequence has $u_1 = 4$ and $u_{n+1} = 2u_n - 1$. Find u_4 .

**

Q17 A sequence has $u_1 = 2$, $u_2 = 5$ and $u_{n+2} = u_{n+1} + u_n$. Find u_6 .

Q18 A geometric sequence has second term 12 and fifth term 324. Its common ratio is positive. Find its first term.

4. Testing terms and comparing sequences

Example Problem 4

The n th term is $2n^2 + n$. Is 136 a term?

The idea

1. Set the n th-term expression equal to the given number and solve for n .
2. A valid position must be a positive whole number.

Worked Solution

$$2n^2 + n - 136 = 0$$

Set the rule equal to 136.

$$(2n + 17)(n - 8) = 0$$

Factorise.

$$n = 8 \text{ or } -\frac{17}{2}$$

Solve each factor.

$$n = 8$$

Only the positive integer is a position, so 136 is the eighth term.

4. Testing terms and comparing sequences

Example Problem 5

The n th term is $u_n = 2n^2 + 3n$. Find an expression for the difference between consecutive terms, $u_{n+1} - u_n$.

The idea

1. Replace every n by $n + 1$ to write the next term.
2. Subtract the whole original expression, using brackets.

Worked Solution

$$u_{n+1} = 2(n+1)^2 + 3(n+1)$$

Substitute $n + 1$ for n .

$$u_{n+1} = 2n^2 + 7n + 5$$

Expand and collect terms.

$$u_{n+1} - u_n = 2n^2 + 7n + 5 - (2n^2 + 3n)$$

Subtract the original term.

$$u_{n+1} - u_n = 4n + 5$$

The quadratic terms cancel.

4 Testing terms and comparing sequences

Q19 The n th term is $n^2 + n + 1$. Is 91 a term? Justify your answer.

Q20 The n th term is $n^2 + 2$. Is 100 a term? Justify your answer.

Q21 Two sequences have n th terms $4n + 7$ and $7n - 5$. At which position are their terms equal?

Q22 Two sequences have n th terms $n^2 + 2n$ and $8n - 5$. At which positions are their terms equal?

Q23 Prove that every term of the sequence $u_n = n^2 + n$ is even for positive integer n .

Q24 The n th term is $u_n = 3n^2 + n$. The difference between two consecutive terms is 100. (a) Find both positions. (b) Find both terms.

Solving principles

1. Label the positions

Use $n = 1$ for the first term unless a different starting index is stated.

2. Check your rule

Substitute several positions, including a term not used to find the rule.

3. Check whether a number is a term

Solve for n . A term occurs only if the solution is a positive whole number.

4. Prove beyond examples

Use an expression for the n th term or a general consecutive difference when a statement is about every term.