

GCSE MATHS

Rearranging formulae

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

The subject stands alone

In $y = 3x + 2$, the subject is y . Making x the subject gives $x = \frac{y-2}{3}$.

Undo operations in reverse order

If x is multiplied by five and then increased by two, subtract two first and divide by five second. Apply each operation to both sides.

A fraction bar groups the numerator

$\frac{a+b}{c}$ divides both a and b by c . It is different from $a + \frac{b}{c}$. Keep brackets around a complete numerator when writing fractions on one line.

Factorise a repeated subject

$ax + bx = x(a + b)$. Gathering all the terms containing the required subject lets you factor it out and then divide.

Powers and roots

For $x^2 = k$ with $k > 0$, $x = \pm\sqrt{k}$; choose the positive root if $x > 0$. A square-root symbol gives a root that is zero or positive, so $y = \sqrt{x}$ requires $y \geq 0$. Keep the whole expression under the root.

1. Inverse operations

Example Problem 1

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Make x the subject: $y = 4x - 7$.

The idea

1. Undo the subtraction of seven.
2. Undo the multiplication by four.

Worked Solution

$$y + 7 = 4x$$

$$x = \frac{y + 7}{4}$$

Add seven to both sides.

Divide the whole expression $y + 7$ by four.

1 Inverse operations

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Q1 Make x the subject: $y = 5x + 8$.

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Q2 Make t the subject: $v = u + at$, where $a \neq 0$.

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Q3 Make r the subject: $P = 2r + 2s$.

*

Q4 Make x the subject: $y = a - bx$, where $b \neq 0$.

*

Q5 Make m the subject: $p = \frac{m}{3} - q$.

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2. Brackets and fractions

Example Problem 2

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Make x the subject: $y = \frac{2x+5}{3}$.

The idea

1. Clear the denominator by multiplying by three.
2. Subtract five, then divide by two.

Worked Solution

$$3y = 2x + 5$$

$$2x = 3y - 5$$

$$x = \frac{3y - 5}{2}$$

Multiply both sides by three.

Subtract five.

Divide by two.

2 Brackets and fractions

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Q6 Make x the subject: $y = 4(x - 3)$.

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Q7 Make x the subject: $y = \frac{3x-7}{5}$.

**

Q8 Make C the subject: $F = \frac{9}{5}C + 32$.

**

Q9 Make t the subject: $s = \frac{(u+v)t}{2}$, where $u + v \neq 0$.

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Q10 Make x the subject: $y = a(bx - c) + d$, where $a, b \neq 0$.

**

3. Powers and roots

Example Problem 3

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Make x the subject: $y = 2x^2 + 7$, where $x > 0$.

The idea

1. Isolate x^2 before taking a square root.
2. Use the positive root because $x > 0$.

Worked Solution

$$2x^2 = y - 7$$

$$x^2 = \frac{y - 7}{2}$$

$$x = \sqrt{\frac{y - 7}{2}}$$

Subtract seven.

Divide by two.

The root covers the entire fraction.

3 Powers and roots

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Q11 Make x the subject: $y = x^2 + 9$, where $x > 0$.

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Q12 Make x the subject: $y = 3x^2 - 5$, where $x > 0$.

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Q13 Make x the subject: $y = \sqrt{2x + 7}$.

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Q14 Make r the subject: $V = \frac{4}{3}\pi r^3$, where $r > 0$.

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Q15 Make x the subject: $y = a\sqrt{x + b} + c$, where $a > 0$.

**

4. The subject appears twice

Example Problem 4

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Make x the subject: $p = 3x + qx$, where $q \neq -3$.

The idea

1. Factor out x so it appears once.
2. Divide by the remaining factor.

Worked Solution

$$p = x(3 + q)$$

$$x = \frac{p}{3 + q}$$

Both terms contain x .

Divide by $3 + q$, not by 3 and q separately.

4 The subject appears twice

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Q16 Make x the subject: $y = ax + bx$, where $a + b \neq 0$.

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Q17 Make x the subject: $y = ax + b - cx$, where $a \neq c$.

**

Q18 Make x the subject: $y = \frac{ax}{2} + \frac{bx}{3}$, where $3a + 2b \neq 0$.

**

Q19 Make x the subject: $a(x + b) = c(x - d)$, where $a \neq c$.

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Q20 Make x the subject: $m(2x - 3) = n(x + 4)$, where $2m \neq n$.

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5. The subject in a denominator

Example Problem 5

★ ★

Make x the subject: $y = \frac{2x+1}{x-3}$, where $y \neq 2$.

The idea

1. Multiply by $x - 3$ to clear the fraction.
2. Collect the x terms, factorise, then divide.

Worked Solution

$$y(x - 3) = 2x + 1$$

Multiply both sides by $x - 3$.

$$yx - 3y = 2x + 1$$

Expand.

$$yx - 2x = 3y + 1$$

Collect x terms on the left.

$$x(y - 2) = 3y + 1$$

Factor out x .

$$x = \frac{3y + 1}{y - 2}$$

Divide by $y - 2$.

5 The subject in a denominator

Q21 Make x the subject: $y = \frac{5}{x+2}$, where $y \neq 0$.

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Q22 Make x the subject: $y = \frac{a}{bx+c}$, where $a, b, y \neq 0$.

**

Q23 Make x the subject: $y = \frac{x+3}{x-2}$, where $y \neq 1$.

**

Q24 Make x the subject: $y = \frac{3x-4}{2x+5}$, where $2y \neq 3$.

**

Q25 Make v the subject: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where $f, u, v \neq 0$ and $u \neq f$.

6. Harder combinations and applications

Example Problem 6

Make x the subject: $y = \frac{3x^2+2}{x^2-1}$, where $x > 0$ and $y \neq 3$.

The idea

1. Treat x^2 as one quantity until it is isolated.
2. Clear the denominator, collect x^2 terms and factorise.
3. Take the positive square root last.

Worked Solution

$$y(x^2 - 1) = 3x^2 + 2$$

Multiply by $x^2 - 1$.

$$yx^2 - 3x^2 = y + 2$$

Expand and collect the x^2 terms.

$$x^2(y - 3) = y + 2$$

Factor out x^2 .

$$x^2 = \frac{y + 2}{y - 3}$$

Divide by $y - 3$.

$$x = \sqrt{\frac{y + 2}{y - 3}}$$

Use the positive square root.

6 Harder combinations and applications

Q26 Make x the subject: $y = \frac{2x+3}{5} - \frac{x-1}{2}$.

**

Q27 Make x the subject: $y = \frac{2x^2+3}{x^2-4}$, where $x > 0$ and $y \neq 2$.

Q28 A cylinder has radius $r > 0$, height h and total surface area $S = 2\pi r^2 + 2\pi r h$. Make h the subject.

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Q29 Make x the subject: $y = \sqrt{\frac{x+3}{x-1}}$, where $y^2 \neq 1$.

Q30 A car travels two equal distances at speeds u and v , without stopping. Its average speed for the whole journey is a . Derive a formula for v in terms of u and a , where $u, v > 0$ and $a < 2u$.

Solving principles

1. Find every occurrence of the subject

For $y = \frac{x+2}{x-3}$, x occurs twice. Clear the denominator before collecting the x terms.

2. Keep whole expressions together

In $y = a(x + b)$, divide by a before subtracting b . In $y = \frac{x+b}{c}$, multiply by c before subtracting b .

3. Choose the simpler side for the subject

From $yx - 2y = 3x + 4$, collect to get $x(y - 3) = 2y + 4$, or $x(3 - y) = -2y - 4$. Both routes work if the signs are kept consistent.

4. Delay a root until the power is isolated

From $y = 3x^2 + 6$, reach $x^2 = \frac{y-6}{3}$ before taking a square root. Do not take roots term by term across an addition.

5. Check by substitution

Substitute your rearranged expression into the original formula and simplify.

6. Check the value you divide by

Division by zero is not defined. In $y = x(a - b)$, the rearrangement $x = \frac{y}{a-b}$ needs $a \neq b$. Use any given sign condition when choosing a root.