

GCSE MATHS

Ratio problems

Worked Solutions

Q1

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$$1.2 \text{ kg} = 1200 \text{ g}$$

Use the same unit for both masses.

$$1200 : 450 = 8 : 3$$

Divide both numbers by 150.

Q2

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$$3 + 4 = 7$$

There are seven parts altogether.

$$56 \div 7 = 8$$

Each part is worth £8.

$$3 \times 8 = 24$$

Multiply by the number of parts in each share.

$$4 \times 8 = 32$$

Q3

**

(a) $21 \div 7 = 3$

Seven parts represent the 21 boys.

$$4 \times 3 = 12$$

Four parts represent the girls.

(b) $\frac{4}{4+7} = \frac{4}{11}$

The fraction uses the whole group as its denominator.

Q4

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$$2 + 5 = 7$$

Red and blue together make seven parts.

$$\text{Blue fraction} = \frac{5}{7}$$

Five of the seven parts are blue.

Q5

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$$\frac{A}{B} = \frac{5}{8}$$

Compare A with B, not with the total.

$$\frac{5}{8} \times 100\% = 62.5\%$$

Convert the fraction to a percentage.

Q6

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$$100\% - 35\% = 65\%$$

The rest of the counters are white.

$$35 : 65 = 7 : 13$$

Divide both parts by five.

Q7

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$$18 \div 3 = 6$$

Three parts represent 18 apples.

$$7 \times 6 = 42$$

Seven parts represent the pears.

$$18 + 42 = 60$$

Add the two counts for the whole box.

Q8

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$$2 + 7 = 9$$

There are nine parts, each of 70 ml.

$$630 \div 9 = 70$$

$$2 \times 70 = 140$$

The concentrate has two parts.

$$7 \times 70 = 490$$

The water has seven parts.

Q9

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$$2 + 3 + 7 = 12$$

Find the value of one part.

$$168 \div 12 = 14$$

$$2 \times 14 = 28$$

Multiply each part count by £14.

$$3 \times 14 = 42$$

$$7 \times 14 = 98$$

Q10

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$$64 \div 8 = 8$$

The longest length represents eight parts.

$$3 \times 8 = 24$$

Find the other lengths.

$$5 \times 8 = 40$$

$$24 + 40 + 64 = 128$$

Add all three lengths.

Q11

**

$$9 - 4 = 5$$

The difference is five parts.

$$35 \div 5 = 7$$

Each part represents 7 cm.

$$4 \times 7 = 28$$

Find the two lengths.

$$9 \times 7 = 63$$

Q12

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$$7 - 2 = 5$$

The difference of five parts is worth £65.

$$65 \div 5 = 13$$

$$2 \times 13 = 26$$

Find the individual amounts.

$$5 \times 13 = 65$$

$$7 \times 13 = 91$$

$$26 + 65 + 91 = 182$$

Add the amounts.

Q13

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$$A : B = 9 : 12$$

Multiply the first ratio by three.

$$B : C = 12 : 10$$

Multiply the second ratio by two so B agrees.

$$A : B : C = 9 : 12 : 10$$

Combine the ratios through their common B value.

Q14

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$$x : y = 8 : 18$$

Use 18 parts for y in both ratios.

$$y : z = 18 : 15$$

$$x : y : z = 8 : 18 : 15$$

Combine the ratios.

$$x : z = 8 : 15$$

Select the required pair.

Q15

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$$R : B : G = 9 : 12 : 10$$

$$9 + 12 + 10 = 31$$

$$155 \div 31 = 5$$

$$R = 45$$

$$B = 60$$

$$G = 50$$

Make the blue parts equal before combining.

The whole box contains 31 parts.

Multiply the ratio by five.

Q16

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$$A : B : C = 8 : 12 : 15$$

$$8 + 12 + 15 = 35$$

$$\frac{A}{A + B + C} = \frac{8}{35}$$

Use 12 for the common B value.

Find the total number of parts.

A contributes eight of the 35 parts.

Q17

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$$B : G : A = 9 : 12 : 2$$

$$9 + 12 + 2 = 23$$

$$184 \div 23 = 8$$

$$A = 2 \times 8 = 16$$

Match the girls at 12 parts.

Each part represents eight people.

Adults make up two parts.

Q18

**

(a) $a + b = 2k + 5k = 7k$

$$c = 3k$$

$$a : b : c = 2 : 5 : 3$$

(b) $\frac{3}{2 + 5 + 3} \times 100\% = 30\%$

Write the first ratio using a common scale k .

The seven parts for $a + b$ already match the second ratio.

Read the three quantities on the same scale.

Compare c with the whole total.

Q19

**

$$R = 2k$$

$$B = 3k$$

$$\frac{2k + 10}{3k} = \frac{4}{3}$$

$$6k + 30 = 12k$$

$$k = 5$$

$$R = 10$$

$$B = 15$$

Use k counters per original part.

Only the red count changes.

Cross-multiply and solve.

Find the original counts, before the addition.

Q20

$$R = 3k$$

$$B = 5k$$

$$\frac{3k - 6}{5k + 10} = \frac{1}{3}$$

$$9k - 18 = 5k + 10$$

$$k = 7$$

$$R = 21$$

$$B = 35$$

Represent the original counts.

Use the changed counts in the new ratio.

Cross-multiply and solve.

Return to the original counts.

Q21

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$$A = 3k$$

$$C = 7k$$

$$\frac{3k}{7k - 12} = \frac{1}{2}$$

$$6k = 7k - 12$$

$$k = 12$$

$$A = 36$$

$$C = 84$$

Represent the original group.

The adults stay and 12 children leave.

Solve for one original part.

Multiply the original ratio by 12.

Q22

**

$$A = 5k$$

$$B = 3k$$

$$5k - 14 = 3k + 14$$

$$2k = 28$$

$$k = 14$$

$$A = 70$$

$$B = 42$$

Represent the original amounts in pounds.

The same transfer decreases one amount and increases the other.

Solve for the original scale.

Find the amounts before the transfer.

Q23

$$A = 3k$$

$$B = 5k$$

$$\frac{3k + 12}{5k + 12} = \frac{5}{7}$$

$$21k + 84 = 25k + 60$$

$$k = 6$$

$$A = 18$$

$$B = 30$$

Represent the present ages.

Both ages increase by 12.

Cross-multiply and solve.

Return to the present ages.

Q24

$$G = 4k$$

$$B = 5k$$

$$\frac{4k + 6}{5k - 6} = \frac{5}{4}$$

$$16k + 24 = 25k - 30$$

$$k = 6$$

$$G + B = 9k = 54$$

Represent the original counts.

Use the new counts.

Cross-multiply and solve.

The question asks for the original total.

Q25

**

$$750 \div 5 = 150$$

$$J = 150$$

$$W = 600$$

$$J_{\text{new}} = 150 + 150 = 300$$

$$300 : 600 = 1 : 2$$

Find one original part, in ml.

Find the original volumes.

Only the juice volume increases.

Simplify the new ratio.

Q26

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$$B = \frac{3}{5} \times 20 = 12$$

$$W = 8$$

$$W_{\text{new}} = 2 \times 12 = 24$$

$$24 - 8 = 16$$

Find the original component volumes.

The unchanged 12 litres of blue is now one part.

Subtract the white paint already present.

Q27

$$200 - 50 = 150$$

$$C = \frac{7}{10} \times 150 = 105$$

$$Z = 45$$

$$Z_{\text{new}} = 45 + 30 = 75$$

$$105 : 75 = 7 : 5$$

Removing a piece of the uniform alloy leaves its component ratio unchanged.

Find the masses in the remaining alloy.

Only zinc is added.

Simplify the final ratio.

Q28

Use 12 ml of each drink

$$A : J = 4, W = 8$$

$$B : J = 9, W = 3$$

$$J = 4 + 9 = 13$$

$$W = 8 + 3 = 11$$

$$J : W = 13 : 11$$

Choose a common multiple of the two total part counts. Any equal volumes give the same ratio.

Find each component in the chosen equal volumes.

Add actual component volumes, not the ratio numbers.

The ratio is already in its simplest form.

Q29

Gold fractions: $\frac{1}{4}$ $\frac{3}{5}$

$$\frac{1}{4}a + \frac{3}{5}b = \frac{1}{2}(a + b)$$

$$5a + 12b = 10a + 10b$$

$$2b = 5a$$

$$a : b = 2 : 5$$

The fractions use the total parts in each alloy.

Let a and b be the alloy masses. Half of the final mass must be gold.

Multiply by 20 to clear the denominators.

Read the ratio in the requested order, A to B .

Q30

Masses: a

$$18 - a$$

$$8a + 14(18 - a) = 10 \times 18$$

$$8a + 252 - 14a = 180$$

$$a = 12$$

$$18 - a = 6$$

Use the known total mass.

Equate the total ingredient cost with the total blend cost.

Expand and solve.

Find the other mass.

Q31

$$R = 2k$$

$$B = 3k$$

$$R_{\text{left}} = \frac{3}{4}(2k) = \frac{3}{2}k$$

$$B_{\text{left}} = \frac{2}{3}(3k) = 2k$$

$$\frac{3}{2}k + 8 = 2k$$

$$k = 16$$

$$R = 32$$

$$B = 48$$

Represent the original counts.

Multiply by the fraction remaining for each colour.

Use the final equality after adding the red counters.

Return to the original counts.

Q32

★★★★

Incomes: $3x, 4x$; spending: $5y, 7y$

$$3x - 5y = 180$$

$$4x - 7y = 180$$

$$x - 2y = 0$$

$$x = 2y$$

$$6y - 5y = 180$$

$$y = 180$$

$$x = 360$$

$$3x = 1080$$

$$4x = 1440$$

Use different scales for the income ratio and the spending ratio.

Income minus spending gives the saving.

Subtract the first equation from the second.

Substitute into the first saving equation.

Find the two incomes.

Q33

★★★★

(a) Combined red = $\frac{7}{15} \times 90 = 42$

$$\frac{3}{5}a + \frac{1}{5}(90 - a) = 42$$

$$3a + 90 - a = 210$$

$$a = 60$$

$$B = 90 - 60 = 30$$

(b) Final blue = $\frac{3}{2} \times 42 = 63$

$$63 - (90 - 42) = 15$$

Use the combined ratio to find the red count.

Let a be the original total in box A; the rest came from B .

Clear the denominator and solve.

Find the other original box total.

Red stays fixed when the extra blue counters are added.

Subtract the 48 blue counters already present.

Q34

$$2(3k + 5k) = 64$$

$$k = 4$$

$$w = 12$$

$$l = 20$$

$$\frac{12 + d}{20 + d} = \frac{2}{3}$$

$$36 + 3d = 40 + 2d$$

$$d = 4$$

$$16 \times 24 - 12 \times 20 = 144$$

The perimeter includes each side twice.

Find the original dimensions.

Let d be the increase in each dimension.

Solve for the increase in length.

Subtract the original area from the new area.

Q35

(a) $A = 5k$

$$B = 4k$$

$$\frac{5k - 18}{4k + 18} = \frac{4}{5}$$

$$25k - 90 = 16k + 72$$

$$k = 18$$

$$A_{\text{original}} = 90$$

$$B_{\text{original}} = 72$$

(b) $A_{\text{final}} = 72$

$$B_{\text{final}} = \frac{2}{3} \times 90 = 60$$

$$72 : 60 = 6 : 5$$

Represent the original volumes.

Subtract the transfer from A and add it to B .

Cross-multiply and solve.

Find the original volumes.

After the transfer, B contains 90 litres; two-thirds remains.

Simplify the final ratio.

Q36

$$G = 3k$$

$$C = 2k$$

$$3k = 2k + 22$$

$$k = 22$$

$$G = 66$$

$$C = 44$$

$$a + b = 110$$

Represent the gold and copper masses before the extra copper is added.

Equal final masses fix the original mixture scale.

The original mixture is 110 g. Let a and b be the masses of alloys A and B .

Q36 continued

$$\frac{2}{5}a + \frac{5}{7}b = 66$$

$$\frac{2}{5}a + \frac{5}{7}(110 - a) = 66$$

$$14a + 25(110 - a) = 2310$$

$$2750 - 11a = 2310$$

$$a = 40$$

$$b = 110 - 40 = 70$$

Use the gold fractions of the two alloys.

Substitute $b = 110 - a$.

Multiply by 35 to clear both denominators.

Solve for the mass of alloy A.

Find the mass of alloy B.