

GCSE MATHS

Ratio problems

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Ratio recap

Equivalent ratios

Multiplying or dividing every entry by the same number keeps the ratio unchanged: $6 : 10 = 3 : 5$. For lengths of 1.2 m and 30 cm, first convert 1.2 m to 120 cm. Their ratio is $120 : 30 = 4 : 1$.

A share and a whole

If red : blue is $2 : 3$, red is $\frac{2}{5}$ of the total and blue is $\frac{3}{5}$. Red is $\frac{2}{3}$ of the blue count. The denominator depends on which quantity you are comparing with.

One common scale

If $A : B = 3 : 5$, write $A = 3k$ and $B = 5k$. Then $A + B = 8k$ and $B - A = 2k$. A given total, share or difference can determine k .

Matching a shared quantity

From $A : B = 2 : 3$ and $B : C = 4 : 5$, use $A : B = 8 : 12$ and $B : C = 12 : 15$. This gives $A : B : C = 8 : 12 : 15$.

1. Ratios and fractions

Example Problem 1

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Two ribbons have lengths of 0.6 m and 45 cm. Write the ratio of the longer length to the shorter in its simplest form, then find the fraction of their combined length made by the shorter ribbon.

The idea

1. Convert both lengths to the same unit, then simplify.
2. Use the combined number of parts as the denominator of the fraction.

Worked Solution

$$0.6 \text{ m} = 60 \text{ cm}$$

Convert to centimetres.

$$60 : 45 = 4 : 3$$

Divide both lengths by 15.

$$\frac{\text{shorter}}{\text{total}} = \frac{3}{4 + 3} = \frac{3}{7}$$

The shorter length is three of the seven combined parts.

1 Ratios and fractions

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- Q1** Two parcels have masses of 1.2 kg and 450 g. Write the ratio of the first mass to the second in its simplest form.

*

- Q2** Share £56 between Amira and Ben in the ratio 3 : 4.

*

- Q3** The ratio of girls to boys in a group is 4 : 7. There are 21 boys. Find (a) the number of girls and (b) the fraction of the group who are girls.

**

- Q4** A bag contains only red and blue counters. The ratio of red counters to blue counters is 2 : 5. What fraction of the counters are blue?

*

- Q5** Two positive quantities are in the ratio $A : B = 5 : 8$. Express A as a percentage of B .

**

- Q6** A bag contains only green and white counters. 35% are green. Write the ratio of green to white counters in its simplest form.

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2. Shares, totals and differences

Example Problem 2

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Two savings amounts are in the ratio 3 : 5. The larger exceeds the smaller by £26. Find both amounts.

The idea

1. Find how many parts separate the two shares.
2. Use the known difference to find one part, then both amounts.

Worked Solution

$$5 - 3 = 2$$

The difference is two parts.

$$26 \div 2 = 13$$

Each part is worth £13.

$$3 \times 13 = 39$$

Multiply by the parts in each share.

$$5 \times 13 = 65$$

$$65 - 39 = 26$$

Check against the given difference.

2 Shares, totals and differences

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Q7 The ratio of the number of apples to the number of pears in a box is 3 : 7. There are 18 apples. How many pears are there, and how many pieces of fruit are there altogether?

**

Q8 A drink is made by mixing concentrate and water in the ratio 2 : 7 by volume. How much concentrate and how much water are needed to make 630 ml of the drink?

**

Q9 Share £168 between Arun, Beth and Cara in the ratio 2 : 3 : 7.

**

Q10 Three lengths are in the ratio 3 : 5 : 8. The longest is 64 cm. Find the other two lengths and the total length.

**

Q11 Two ribbons have lengths in the ratio 4 : 9. The longer ribbon is 35 cm longer than the shorter ribbon. Find the length of each ribbon.

**

Q12 Three savings amounts are in the ratio 2 : 5 : 7. The largest exceeds the smallest by £65. Find all three amounts and their total.

**

3. Combining ratios

Example Problem 3

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Positive quantities satisfy $a : b = 3 : 4$ and $b : c = 10 : 7$. Their total is 147. Find a , b and c .

The idea

1. Make the two values of the shared quantity agree.
2. Use the three-part ratio and the given total to find the quantities.

Worked Solution

$$a : b = 15 : 20$$

$$b : c = 20 : 14$$

$$a : b : c = 15 : 20 : 14$$

$$147 \div (15 + 20 + 14) = 3$$

$$a = 45$$

$$b = 60$$

$$c = 42$$

Multiply the first ratio by five and the second by two to make b equal to 20 in both.

Now the three quantities share one scale.

Find the value of one part.

Multiply the three-part ratio by three.

3 Combining ratios

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Q13 Three positive quantities satisfy $A : B = 3 : 4$ and $B : C = 6 : 5$. Find $A : B : C$ in its simplest form.

**

Q14 Positive quantities satisfy $x : y = 4 : 9$ and $y : z = 6 : 5$. Find $x : z$ in its simplest form.

**

Q15 A box contains only red, blue and green counters. Red : blue is $3 : 4$ and blue : green is $6 : 5$. There are 155 counters altogether. Find the number of each colour.

**

Q16 Positive quantities satisfy $A : B = 2 : 3$ and $B : C = 4 : 5$. What fraction of $A + B + C$ is A ?

**

Q17 A trip has boys, girls and adults. Boys : girls is $3 : 4$ and girls : adults is $6 : 1$. There are 184 people altogether. How many are adults?

**

Q18 Positive quantities satisfy $a : b = 2 : 5$ and $(a + b) : c = 7 : 3$. Find (a) $a : b : c$ and (b) the percentage of $a + b + c$ represented by c .

**

4. Changing ratios

Example Problem 4

Red and blue counters are in the ratio 4 : 7. Eight blue counters are removed and the ratio becomes 2 : 3. Find the original counts.

The idea

1. Write the original counts with one common scale.
2. Apply the change to the blue count and use the new ratio to form an equation.

Worked Solution

$$R = 4k$$

$$B = 7k$$

$$\frac{4k}{7k - 8} = \frac{2}{3}$$

$$12k = 14k - 16$$

$$k = 8$$

$$R = 32$$

$$B = 56$$

$$32 : (56 - 8) = 32 : 48 = 2 : 3$$

The original counts are multiples of four and seven.

Only the blue count is reduced.

Cross-multiply and solve.

Find the original counts.

Check the changed ratio.

4 Changing ratios

- Q19** A bag contains red and blue counters in the ratio 2 : 3. Ten red counters are added, making the ratio 4 : 3. Find the original number of each colour.

**

- Q20** A bag contains red and blue counters in the ratio 3 : 5. Six red counters are removed and ten blue counters are added. The ratio of red to blue counters is now 1 : 3. Find the original number of each colour.

- Q21** Adults and children are in the ratio 3 : 7. Twelve children leave, and the ratio becomes 1 : 2. Find the original numbers of adults and children.

**

Q22 Ali and Bea have money in the ratio 5 : 3. Ali gives Bea £14, after which they have equal amounts. How much did each have originally? **

Q23 Two people have ages in the ratio 3 : 5. In 12 years their ages will be in the ratio 5 : 7. Find their present ages. ***

Q24 Girls and boys are in the ratio 4 : 5. Six girls join and six boys leave. The new ratio is 5 : 4. How many pupils were in the group originally? ***

5. Mixtures

Example Problem 5

Paint *A* contains red and white paint in the ratio 1 : 3. Paint *B* contains red and white paint in the ratio 2 : 1. In what volume ratio should *A* and *B* be mixed to give equal amounts of red and white?

The idea

1. Convert each red-to-white ratio into a red fraction of that paint.
2. Equate the total red volume to half of the combined volume.

Worked Solution

Red fractions: $\frac{1}{4}$
 $\frac{2}{3}$

$$\frac{1}{4}a + \frac{2}{3}b = \frac{1}{2}(a + b)$$

$$3a + 8b = 6a + 6b$$

$$2b = 3a$$

$$a : b = 2 : 3$$

$$\frac{1}{4}(2) + \frac{2}{3}(3) = \frac{5}{2}$$

Convert each component ratio to a fraction of its own whole.

Let *a* and *b* be the paint volumes. Half of the mixture must be red.

Multiply by 12.

Write the required ratio in the order *A* to *B*.

With 2 litres of *A* and 3 litres of *B*, red is half of the 5 litres.

5 Mixtures

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- Q25** A 750 ml drink contains juice and water in the ratio 1 : 4. A further 150 ml of juice is added. Find the new ratio of juice to water.

★★

- Q26** Twenty litres of paint contains only blue and white paint in the ratio 3 : 2. How much white paint must be added to make the ratio 1 : 2?

★★

- Q27** A uniform 200 g alloy contains only copper and zinc in the ratio 7 : 3. A 50 g piece of the alloy is removed. Then 30 g of zinc is added to the remainder. Find the final ratio of copper to zinc.

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- Q28** Drink A contains juice and water in the ratio 1 : 2. Drink B contains juice and water in the ratio 3 : 1. Equal volumes of the two drinks are mixed. Find the ratio of juice to water in the mixture. ***

- Q29** Alloy A contains only gold and copper in the ratio 1 : 3 by mass. Alloy B contains only gold and copper in the ratio 3 : 2. In what mass ratio should A and B be mixed to obtain equal masses of gold and copper? ****

- Q30** Tea A costs £8 per kg and tea B costs £14 per kg. A shop makes an 18 kg blend costing £10 per kg, with no other costs. Find the mass of each tea used. ***

6. Problems with several steps

Example Problem 6

Water volumes in tanks A and B are in the ratio $3 : 2$. Twelve litres are transferred from A to B , making the volumes equal. A quarter of the water now in B is then used. Find the final ratio $A : B$.

The idea

1. Subtract the transfer from A and add it to B .
2. Find the volumes after the transfer before applying the fraction used.

Worked Solution

$$A = 3k$$

Represent the original volumes.

$$B = 2k$$

$$3k - 12 = 2k + 12$$

The transfer reduces A and increases B .

$$k = 24$$

$$A_{\text{after}} = 72 - 12 = 60$$

Find the volumes after the transfer.

$$B_{\text{after}} = 48 + 12 = 60$$

$$B_{\text{final}} = \frac{3}{4} \times 60 = 45$$

Three-quarters of B remains.

$$A : B = 60 : 45 = 4 : 3$$

A stays at 60 litres during the final step.

6 Problems with several steps

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- Q31** A bag contains red and blue counters in the ratio $2 : 3$. A quarter of the red counters and a third of the blue counters are removed. Eight red counters are then added, leaving equal numbers of red and blue. Find the original counts.

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- Q32** Amir and Bella have monthly incomes in the ratio $3 : 4$ and monthly spending in the ratio $5 : 7$. Each saves £180 per month. Find each person's monthly income.

★★★★

- Q33** Box A contains red and blue counters in the ratio $3 : 2$. Box B contains red and blue counters in the ratio $1 : 4$. All 90 counters are combined, giving a ratio of $7 : 8$. Find (a) the original total in each box and (b) how many blue counters must then be added to make the ratio $2 : 3$.

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- Q34** A rectangle has width : length = 3 : 5 and perimeter 64 cm. Both dimensions are increased by the same amount, making width : length = 2 : 3. Find the increase in area. ***

- Q35** Water volumes in tanks *A* and *B* are in the ratio 5 : 4. Transferring 18 litres from *A* to *B* changes the ratio to 4 : 5. A third of the water now in *B* is then used. Find (a) the original volume in each tank and (b) the final ratio *A* : *B*. ***

- Q36** Alloy *A* contains gold and copper in the ratio 2 : 3 by mass. Alloy *B* contains gold and copper in the ratio 5 : 2. The alloys are mixed to give a gold-to-copper ratio of 3 : 2. Adding 22 g of copper then makes this ratio 1 : 1. Find the mass of each alloy used. ****

Solving principles

1. Match the given amount to the right parts

For $A : B = 3 : 5$, the whole is eight parts and the difference is two parts. A value for A represents three parts. Use the quantity actually given to find one part.

2. Make changes to amounts

For original counts $3k$ and $5k$, removing six from the first and adding ten to the second gives $3k - 6$ and $5k + 10$. Form the new ratio from these expressions.

3. Keep track of each stage

A transfer of t from A to B changes the amounts to $A - t$ and $B + t$. Apply any later percentage or fraction to the amount at that later stage.

4. Use different scales for independent ratios

An income ratio $3 : 4$ can give $3x$ and $4x$, while a spending ratio $5 : 7$ gives $5y$ and $7y$. Connect the two scales using the information about savings.

5. Mix component amounts

A mixture in the ratio $1 : 3$ contains one-quarter of its first component. If its total mass is a , that component has mass $\frac{1}{4}a$. Calculate each mixture's contribution before adding them.

6. Work backwards from the last change

If adding a known amount makes two components equal, that amount was the difference beforehand. Use this to recover the mixture's original total before finding how it was made.