

GCSE MATHS

Quadratic equations

Worked Solutions

Q1

*

$$(x - 3)(x - 4) = 0$$

$$x = 3 \text{ or } 4$$

Find two numbers with product 12 and sum -7.

Set each factor equal to zero.

Q2

**

$$(x + 6)(x - 4) = 0$$

$$x = -6 \text{ or } 4$$

Six and -4 have product -24 and sum two.

Set each factor equal to zero.

Q3

**

$$3x(x - 4) = 0$$

$$x = 0 \text{ or } 4$$

Factor out $3x$.

Either factor can be zero; do not divide by x .

Q4

**

$$(2x - 5)(x + 3) = 0$$

$$2x = 5 \text{ or } x = -3$$

$$x = \frac{5}{2} \text{ or } -3$$

The middle terms are $6x - 5x$.

Set each factor to zero.

Divide the first equation by two.

Q5

**

$$(3x - 5)(3x + 5) = 0$$

$$3x = 5 \text{ or } -5$$

$$x = \pm \frac{5}{3}$$

Use the difference of two squares.

Set each factor to zero.

Divide by three.

Q6

**

$$x^2 + 3x - 28 = 0$$

Expand and subtract 28.

$$(x + 7)(x - 4) = 0$$

Factorise.

$$x = -7 \text{ or } 4$$

Use the zero-product rule.

Q7

**

$$a = 1$$

Read the coefficients, including their signs.

$$b = -3$$

$$c = -1$$

$$x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

Substitute into the formula.

$$x = \frac{3 \pm \sqrt{13}}{2}$$

Simplify the expression under the square root.

Q8

**

$$x = \frac{-5 \pm \sqrt{25 + 32}}{4}$$

Use $a = 2$, $b = 5$ and $c = -4$.

$$x = \frac{-5 \pm \sqrt{57}}{4}$$

Simplify inside the root.

$$x = 0.637 \text{ or } -3.137$$

Evaluate each branch and round at the end.

Q9

**

$$x = \frac{4 \pm \sqrt{16 + 24}}{6}$$

Substitute into the formula.

$$x = \frac{4 \pm 2\sqrt{10}}{6}$$

Simplify the surd.

$$x = \frac{2 \pm \sqrt{10}}{3}$$

Divide numerator and denominator by two.

Q10

**

$$2x^2 + 4x + 7 = 2(x + 1)^2 + 5$$

Complete the square.

$$2(x + 1)^2 + 5 \geq 5 > 0$$

The expression cannot equal zero.

Q11

$$5x^2 - 3x - 6 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 120}}{10}$$

$$x = 1.44 \text{ or } -0.84$$

Collect terms on one side.

Use $a = 5$, $b = -3$, $c = -6$.

Evaluate and round.

Q12

Extension — beyond GCSE

$$k^2 - 64 = 0$$

$$k^2 = 64$$

$$k = \pm 8$$

For a repeated root, the expression under the square root is zero.

Add 64.

Include both square roots.

Q13

**

$$x^2 + 8x + 3 = (x + 4)^2 - 16 + 3$$

$$(x + 4)^2 - 13$$

Halve eight and compensate for the added square.

Collect constants.

Q14

**

$$(a) \quad x^2 - 6x + 11 = (x - 3)^2 + 2$$

$$(b) \quad (x - 3)^2 \geq 0$$

$$\text{min} = 2 \text{ at } x = 3$$

Halve -6 and adjust the constant.

A square cannot be negative.

Set the square equal to zero.

Q15

**

$$(x + 2)^2 - 11 = 0$$

$$(x + 2)^2 = 11$$

$$x = -2 \pm \sqrt{11}$$

Complete the square.

Add 11.

Take both square roots and subtract two.

Q16

$$2(x^2 - 6x) + 7$$

Factor two from the x terms.

$$2((x - 3)^2 - 9) + 7$$

Complete the square inside the bracket.

$$2(x - 3)^2 - 11$$

Multiply the adjustment by two.

Q17

$$-x^2 + 10x - 18 = -(x - 5)^2 + 7$$

Complete the square, keeping the minus sign.

$$-(x - 5)^2 \leq 0$$

The negative of a square is at most zero.

$$\max = 7 \text{ at } x = 5$$

Set the square to zero.

Q18

$$2(x - 2)^2 - 7 = 0$$

Complete the square.

$$(x - 2)^2 = \frac{7}{2}$$

Add seven and divide by two.

$$x = 2 \pm \frac{\sqrt{14}}{2}$$

Take both roots and add two.

Q19

$$x(x + 5) = 84$$

Form an area equation.

$$(x + 12)(x - 7) = 0$$

Expand, rearrange and factorise.

$$x = 7$$

Reject -12 because a length is positive.

$$P = 2(7 + 12) = 38 \text{ cm}$$

Use both dimensions.

Q20

$$n(n + 1) = 156$$

Let n be the smaller integer.

$$(n + 13)(n - 12) = 0$$

Expand, rearrange and factorise.

$$n = 12$$

Reject the negative root.

$$n + 1 = 13$$

Find the next integer.

Q21

$$t(12 - 5t) = 0$$

$$t = 0 \text{ or } \frac{12}{5}$$

$$t = 2.4 \text{ s}$$

Set $h = 0$ and factorise.

Set each factor equal to zero.

The root $t = 0$ is launch, not return.

Q22

$$l = 16 - w$$

$$w(16 - w) = 55$$

$$(w - 5)(w - 11) = 0$$

Dimensions: 5 cm by 11 cm

Use half the perimeter.

Use the area.

Rearrange and factorise.

The two roots give the length and width in either order.

Q23

$$A = x(20 - x)$$

$$A = 100 - (x - 10)^2$$

$$(x - 10)^2 \leq 4$$

$$-2 \leq x - 10 \leq 2$$

$$8 \leq x \leq 12$$

The other side is $20 - x$.

Complete the square.

Use A at least 96.

Take the interval between the square roots.

Add ten throughout.

Q24

Extension — beyond GCSE

$$4k^2 - 4(k + 6) = 0$$

$$k^2 - k - 6 = 0$$

$$(k - 3)(k + 2) = 0$$

$$x = k$$

$$(k, x) = (3, 3) \text{ or } (-2, -2)$$

Set $b^2 - 4ac$ equal to zero.

Divide by four.

Factorise to find k .

The repeated root is $-b/(2a)$.

Match the root to each value of k .