

GCSE MATHS

Quadratic equations

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Zero product

If $AB = 0$, then $A = 0$ or $B = 0$. Factorise only after collecting all terms on one side.

Quadratic formula

For $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Keep the sign of each coefficient.

Completing the square

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}.$$

Number of solutions

The square of a number cannot be negative. For example, $(x - 2)^2 = -3$ has no solutions; $(x - 2)^2 = 0$ has one.

A calculator may be used. Give exact answers unless a question asks for rounding.

1. Solving by factorising

Example Problem 1

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Solve $2x^2 - 7x + 3 = 0$.

The idea

1. Split -7 into -6 and -1; their product is the product of the first and last coefficients.
2. Factorise in pairs, then solve each factor.

Worked Solution

$$2x^2 - 6x - x + 3 = 0$$

Split $-7x$ using numbers with product six.

$$2x(x - 3) - (x - 3) = 0$$

Factorise each pair.

$$(2x - 1)(x - 3) = 0$$

Take out the common bracket.

$$x = \frac{1}{2} \text{ or } 3$$

Set each factor to zero.

1 Solving by factorising

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Q1 Solve $x^2 - 7x + 12 = 0$.

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Q2 Solve $x^2 + 2x - 24 = 0$.

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Q3 Solve $3x^2 - 12x = 0$.

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Q4 Solve $2x^2 + x - 15 = 0$.

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Q5 Solve $9x^2 - 25 = 0$.

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Q6 Solve $x(x + 3) = 28$.

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2. Quadratic formula and roots

Example Problem 2

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Solve $2x^2 - 4x - 3 = 0$, giving exact answers.

The idea

1. Use the formula because integer factorisation is unavailable.
2. Keep the square root exact and simplify both numerator terms.

Worked Solution

$$a = 2$$

Identify the signed coefficients.

$$b = -4$$

$$c = -3$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{4}$$

Substitute into the quadratic formula.

$$x = \frac{4 \pm 2\sqrt{10}}{4}$$

Simplify the root.

$$x = \frac{2 \pm \sqrt{10}}{2}$$

Cancel a common factor of two.

2. Quadratic formula and roots

Example Problem 3

Extension — beyond GCSE

The equation $x^2 + kx + 9 = 0$ has a repeated root. Find the possible values of k and the corresponding roots.

The idea

1. In the quadratic formula, the $\pm$ choices give the same root when $b^2 - 4ac = 0$.
2. Set $b^2 - 4ac = 0$, solve for k , then find the repeated root.

Worked Solution

$$k^2 - 4(1)(9) = 0$$

$$k^2 = 36$$

$$k = 6 \text{ or } -6$$

$$x = \frac{-k}{2}$$

$$k = 6 : x = -3$$

$$k = -6 : x = 3$$

Use $a = 1$, $b = k$ and $c = 9$.

Take both square roots.

The square-root term in the formula is zero.

Match each root to its value of k .

2 Quadratic formula and roots

Q7 Solve $x^2 - 3x - 1 = 0$. Give exact answers.

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Q8 Solve $2x^2 + 5x - 4 = 0$. Give answers to 3 decimal places.

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Q9 Solve $3x^2 - 4x - 2 = 0$. Give exact answers.

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Q10 How many real solutions does $2x^2 + 4x + 7 = 0$ have? Justify your answer.

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Q11 Solve $5x^2 = 3x + 6$. Give answers to 2 decimal places.

Q12 Extension — beyond GCSE

The equation $x^2 + kx + 16 = 0$ has one repeated root. Find all possible values of k .

3. Completing the square

Example Problem 4

Find the minimum value of $3x^2 - 12x + 8$.

The idea

1. Factor out three before completing the square.
2. Use the fact that a square is non-negative.

Worked Solution

$$3(x^2 - 4x) + 8$$

Factor three from the x terms.

$$3((x - 2)^2 - 4) + 8$$

Complete the square inside the bracket.

$$3(x - 2)^2 - 4$$

Multiply out the constant adjustment.

$$\text{min} = -4 \text{ at } x = 2$$

The square has minimum zero.

3 Completing the square

Q13 Write $x^2 + 8x + 3$ in the form $(x + a)^2 + b$.

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Q14 (a) Write $x^2 - 6x + 11$ in completed-square form. (b) State its minimum value.

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Q15 Solve $x^2 + 4x - 7 = 0$ by completing the square.

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Q16 Write $2x^2 - 12x + 7$ in completed-square form.

Q17 Find the maximum value of $-x^2 + 10x - 18$ and the value of x where it occurs.

Q18 Solve $2x^2 - 8x + 1 = 0$ by completing the square. Give exact answers.

4. Quadratic models and reasoning

Example Problem 5

A rectangle has length 3 cm more than its width and area 70 cm^2 . Find its dimensions.

The idea

1. Define the width and write the length using the same variable.
2. Solve the area equation, then use the positive root.

Worked Solution

$$w(w + 3) = 70$$

Write the area in terms of the width.

$$w^2 + 3w - 70 = 0$$

Expand and subtract 70.

$$(w + 10)(w - 7) = 0$$

Factorise.

$$w = 7 \text{ cm}$$

Reject the negative width.

$$l = 10 \text{ cm}$$

4 Quadratic models and reasoning

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Q19 A rectangle has width x cm and length $(x + 5)$ cm. Its area is 84 cm^2 . Find its perimeter.

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Q20 The product of two consecutive positive integers is 156. Find the integers.

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Q21 A ball's height in metres is $h = 12t - 5t^2$, where t is seconds after launch. When does it return to height zero?

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Q22 A rectangle has perimeter 32 cm and area 55 cm^2 . Find its dimensions.

Q23 A rectangle has perimeter 40 cm. Its area is at least 96 cm^2 . Find the possible values of its width x cm.

Q24 Extension — beyond GCSE

The equation $x^2 - 2kx + k + 6 = 0$ has a repeated root. Find all possible values of k and the corresponding roots.

Solving principles

1. Rearrange first

The zero-product rule does not apply to an expression equal to a non-zero number.

2. Keep both roots

Include the plus and minus branches. A context may rule one out after solving.

3. Use exact values

Keep fractions and surds through the working. Round only when asked.

4. Interpret a square

A square is at least zero. Completed-square form reveals a minimum, maximum or repeated root.