

GCSE MATHS

Quadratic and other curved graphs

Worked Solutions

Q1

★

$$y = (-2)^2 - 3 = 1$$

Square the signed input.

Q2

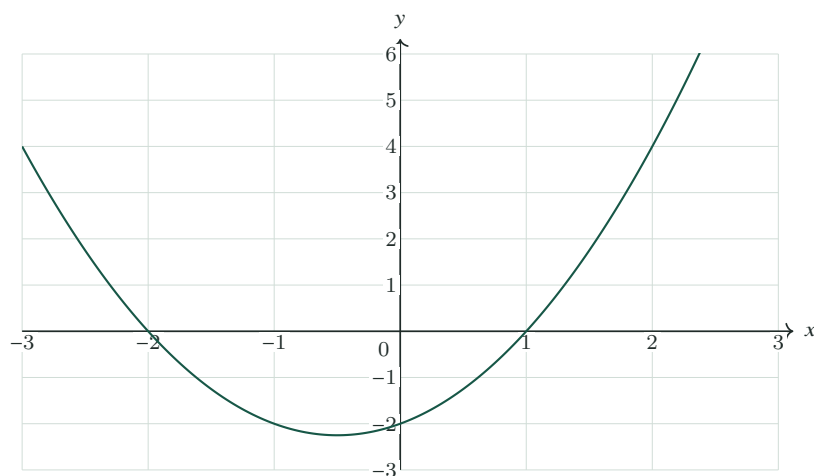
★★

$$y = 0, -2, -2, 0, 4$$

Substitute each x in order.

Sketch shown below

Plot the key points and join with the appropriate shape.

**Q3**

★★

$$x = -3$$

Read where the curve crosses the x -axis.

$$x = 1$$

Q4

**

$$x + 2 = 0 \Rightarrow x = -2$$

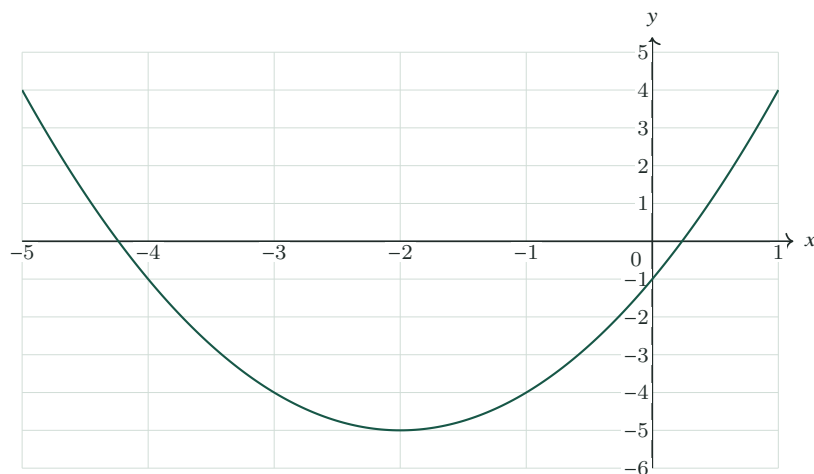
The square is zero at the minimum.

$$y = -5$$

Substitute into the completed-square form.

Sketch shown below

Plot the key points and join with the appropriate shape.



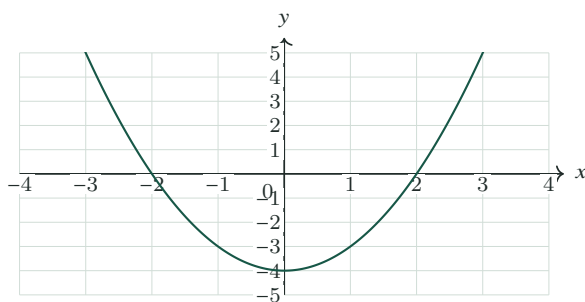
Q5

$$x^2 - 4 = 0 \Rightarrow x = \pm 2$$

Find the x -intercepts.

Minimum $(0, -4)$

Draw a smooth U -shaped curve through the labelled points.



Q6

$$y = (x + 3)^2 + 4$$

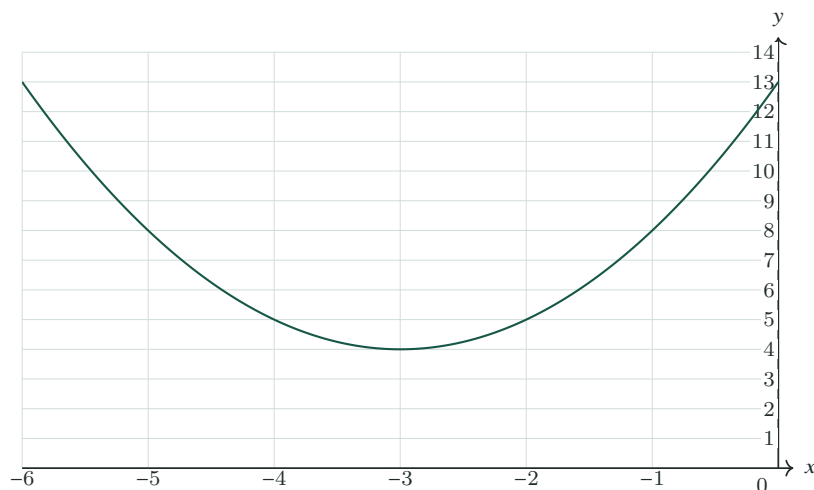
$$y_{\min} = 4$$

Sketch shown below

Complete the square.

The square cannot be negative.

Plot the key points and join with the appropriate shape.

**Q7**

**

$$y = (-2)^3 - 2 = -8 - 2 = -10$$

A negative input has a negative cube.

Q8

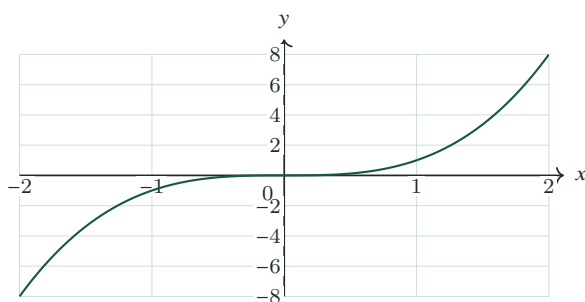
**

$$y = -8, -1, 0, 1, 8$$

Smooth increasing curve

Cube the five integer inputs.

Join the points and flatten near the origin.

**Q9**

**

$$y = 0 \text{ and } x = 0$$

These are the horizontal and vertical asymptotes.

Q10

**

$$6/(-3) = -2$$

Divide 6 by each negative input.

$$6/(-2) = -3$$

$$6/1 = 6$$

Repeat for the positive inputs.

$$6/3 = 2$$

Q11

**

$$x^3 + 8 = 0 \Rightarrow x^3 = -8$$

Set y to zero.

$$x = -2$$

Take the cube root.

Q12

$$-4 = k/3 \Rightarrow k = -12$$

Multiply both sides by 3.

Q13

**

$$3^{-2} = \frac{1}{9}$$

Use reciprocals for negative powers.

$$3^{-1} = \frac{1}{3}$$

$$3^0 = 1$$

Calculate the non-negative powers.

$$3^1 = 3$$

$$3^2 = 9$$

Q14

**

$$x = 0 \Rightarrow y = a \times 1 = a$$

The y-intercept gives a directly.

$$a = 3$$

Read the y-intercept from the graph.

Q15

**

$$2^x = 8 = 2^3$$

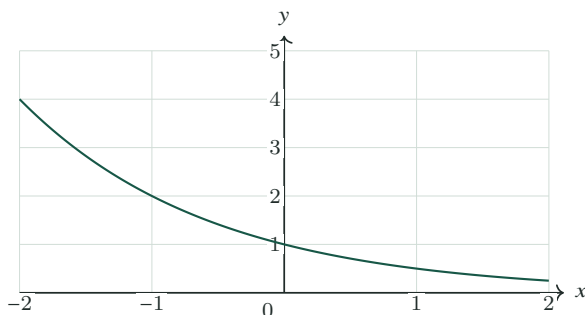
Divide by 3 and recognise the power.

$$x = 3$$

Equal powers of 2 have equal indices.

Q16

$$y = 4, 2, 1, \frac{1}{2}, \frac{1}{4}$$

Calculate the y -values in order.Decreasing curve above the x -axisJoin smoothly; it approaches zero as x increases.

Q17

$$k^3 = 27 \Rightarrow k = 3$$

Substitute the point and take the positive cube root.

Q18

$$x = 0 \Rightarrow y = 1$$

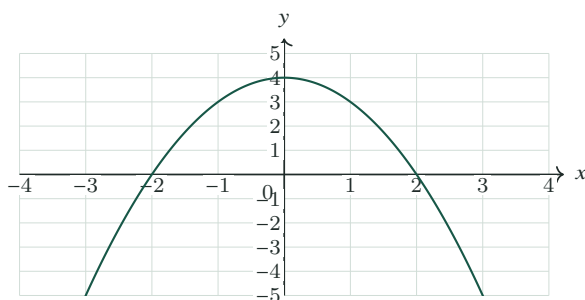
$$x = 1 \Rightarrow y = 2$$

Both equations give these coordinates.

Q19

$$-x^2 + 4 = 0 \Rightarrow x = \pm 2$$

Find the roots.

Maximum $(0, 4)$ The negative square makes an inverted U .

Q20

★★★★

$$x^2 - 2x - 3 = 0$$

Set the y -expressions equal.

$$(x - 3)(x + 1) = 0 \Rightarrow x = 3, -1$$

Factorise and solve.

$$(-1, 1)$$

Find y at each x .

$$(3, 9)$$

Q21

★★★

$$(x - 1)^2 \geq 0$$

The square is non-negative.

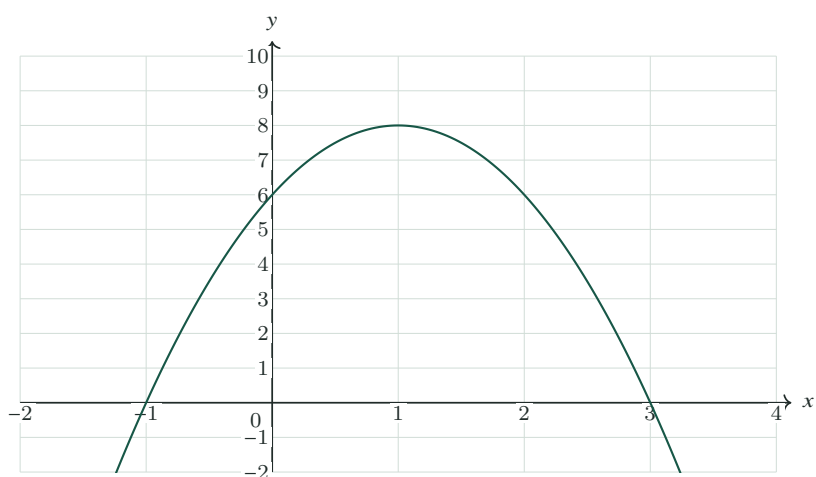
$$y \leq 8$$

The maximum occurs when the square is zero.

$$x = 1$$

Sketch shown below

Plot the key points and join with the appropriate shape.



Q22

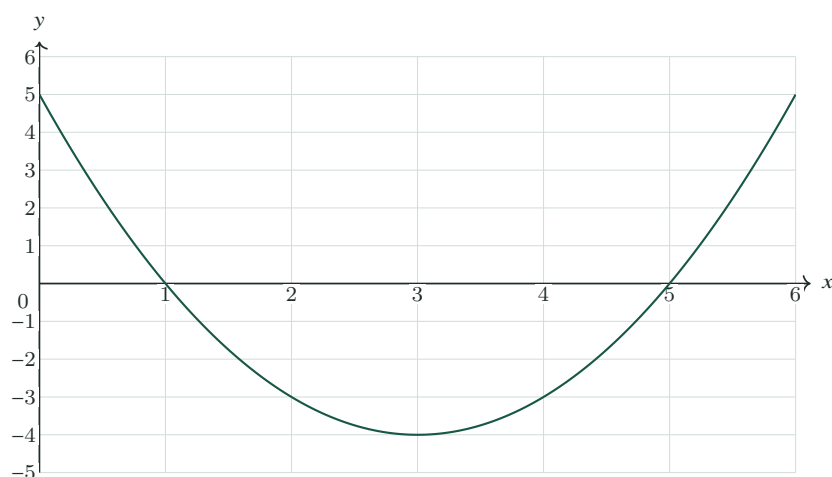
$$a = 3$$

$$b = -4$$

Sketch shown below

Read the horizontal and vertical coordinates of the minimum.

Plot the key points and join with the appropriate shape.



Q23

$$y = (x + 1)^2 + c - 1$$

$$c - 1 > 0 \Rightarrow c > 1$$

Complete the square.

The minimum must lie strictly above the x -axis.

Q24

$$6 = x^2 + x \Rightarrow x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0 \Rightarrow x = -3, 2$$

$$(-3, -2)$$

$$(2, 3)$$

Multiply by non-zero x .

Factorise and solve.

Substitute into $y = x + 1$.