

GCSE MATHS

Quadratic and other curved graphs

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Quadratics

$y = ax^2 + bx + c$ has a vertical line of symmetry. Roots are x -intercepts; the y -intercept is c . Completing the square reveals the turning point.

Cubics and reciprocals

$y = x^3$ passes through the origin. For $y = k/x$, x cannot be zero; its branches approach the axes without meeting them. Such a line is called an asymptote.

Exponentials

$y = k^x$ passes through $(0, 1)$. It grows when $k > 1$ and decreases when $0 < k < 1$.

Intersections

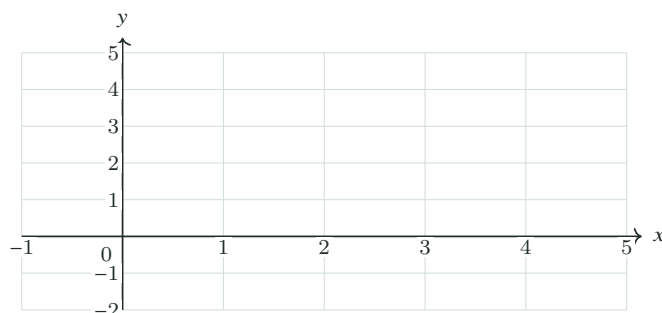
At an intersection, both graphs have the same x and y . Reading a graph gives estimates; solving the equations can give exact coordinates.

A calculator may be used. Give exact values unless an estimate is requested.

1. Quadratic graphs

Example Problem 1

Find the turning point and roots of $y = x^2 - 4x + 3$. Sketch the graph, labelling these features.



The idea

1. Complete the square for the turning point; factorise for the roots.

Worked Solution

$$y = (x - 2)^2 - 1$$

Complete the square.

Turning point $(2, -1)$

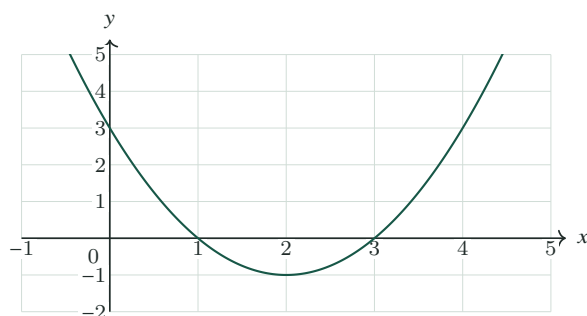
The square has minimum zero.

$$y = (x - 1)(x - 3)$$

Factorise.

$$x = 1 \text{ or } x = 3$$

Set each factor to zero.



1 Quadratic graphs

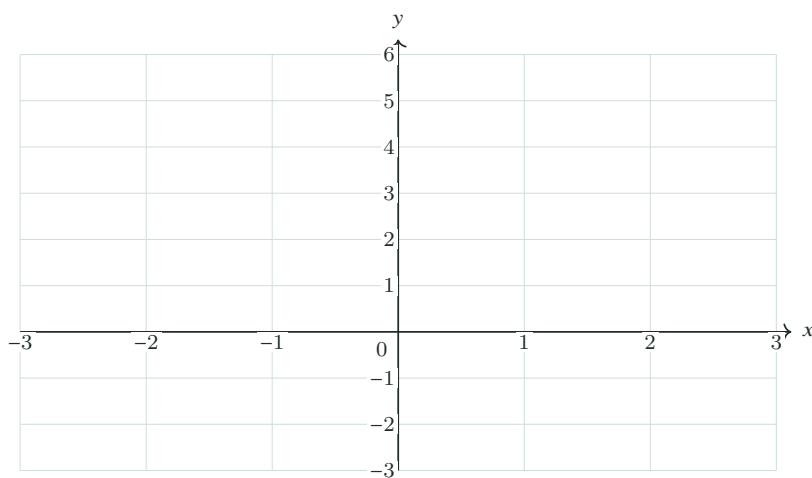
Q1 For $y = x^2 - 3$, find y when $x = -2$.

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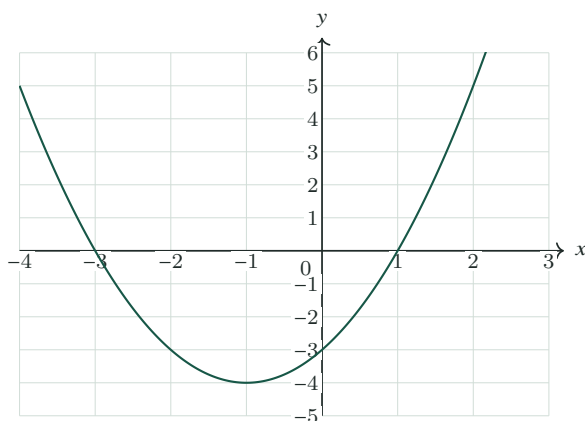
Q2 Complete the table for $y = x^2 + x - 2$. Draw the graph on the axes.

**

x	-2	-1	0	1	2
y					

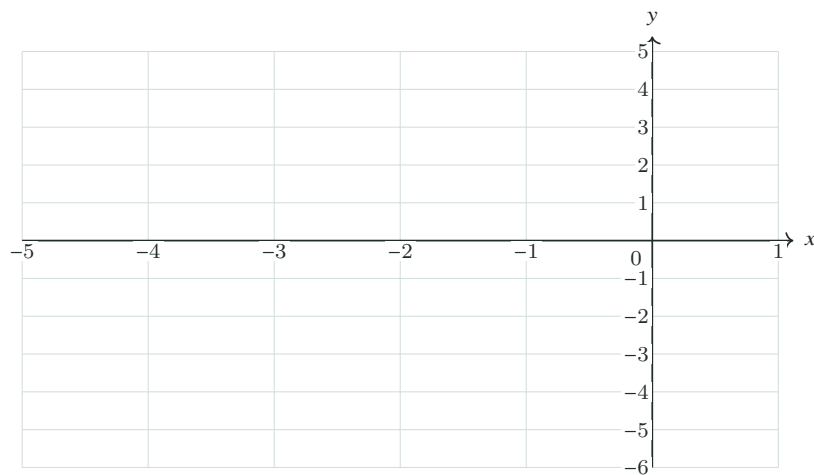
**Q3** Read the x -intercepts of the graph.

**

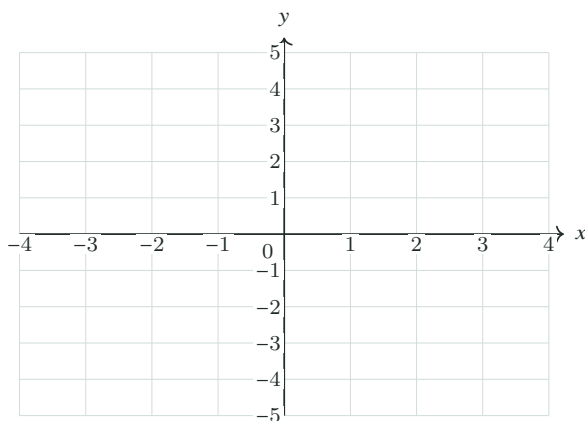


Q4 Find the turning point of $y = (x + 2)^2 - 5$. Sketch the graph on the axes.

**



Q5 Sketch $y = x^2 - 4$ on the grid. Label both roots and the turning point.



Q6 Find the minimum value of $y = x^2 + 6x + 13$. Sketch the graph on the axes.



2. Cubic and reciprocal graphs

Example Problem 2

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Find points on $y = 4/x$ for $x = -4, -2, -1, 1, 2, 4$.

The idea

1. Divide 4 by each x -value; leave a gap at $x = 0$.

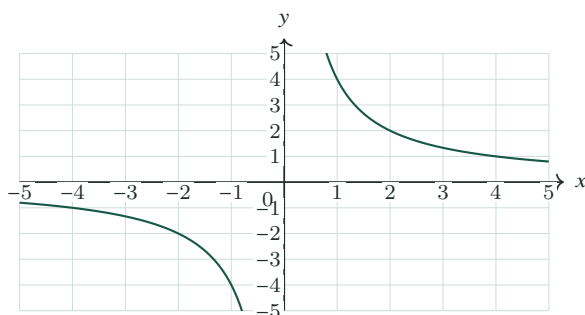
Worked Solution

$$y = -1, -2, -4, 4, 2, 1$$

Calculate the corresponding y -values.

$$x \neq 0$$

The curve has two separate branches.



2 Cubic and reciprocal graphs

Q7 For $y = x^3 - 2$, find y when $x = -2$.

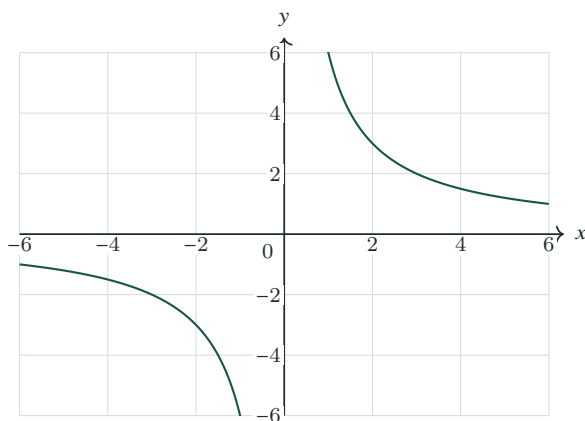
**

Q8 Sketch $y = x^3$ for $-2 \leq x \leq 2$. Mark the points for $x = -2, -1, 0, 1, 2$.

**



Q9 The graph is $y = 6/x$. State the two lines its branches approach. **



Q10 Find y for $y = 6/x$ at $x = -3, -2, 1, 3$. **

Q11 Find the x -intercept of $y = x^3 + 8$. **

Q12 The point $(3, -4)$ lies on $y = k/x$. Find k . ***

3. Exponential graphs

Example Problem 3

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Calculate five points on $y = 2^x$ for $x = -2, -1, 0, 1, 2$.

The idea

1. Use reciprocal powers for negative indices.

Worked Solution

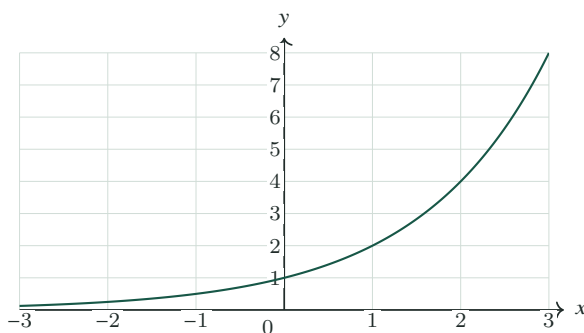
$$2^{-2} = 1/4$$

Negative powers give reciprocals.

$$2^{-1} = 1/2$$

$$y = 0.25, 0.5, 1, 2, 4$$

List values in the given x -order.



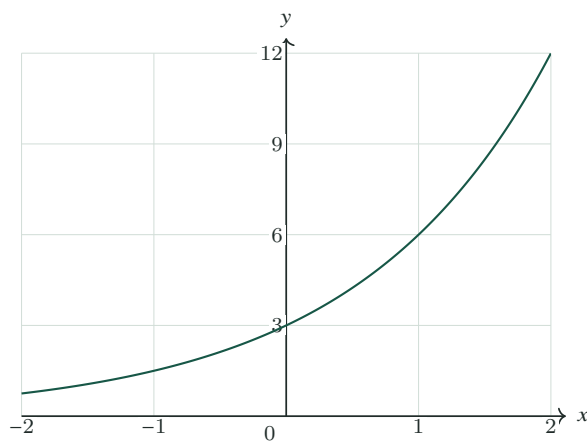
3 Exponential graphs

Q13 Find the values of 3^x for $x = -2, -1, 0, 1, 2$.

**

Q14 The curve has equation $y = a \times 2^x$. Read the y -intercept and hence find a .

**



Q15 Find x if $3 \times 2^x = 24$.

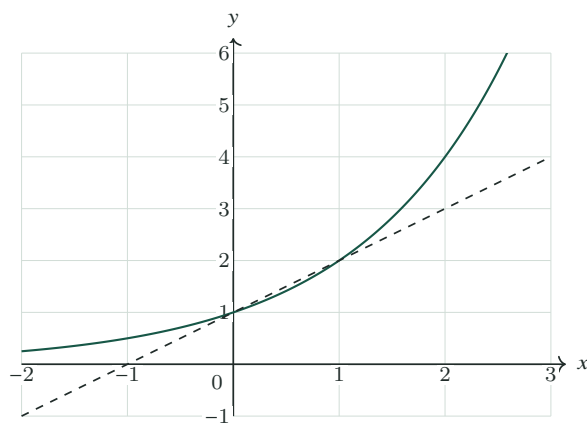
**

Q16 Sketch $y = \left(\frac{1}{2}\right)^x$ for $-2 \leq x \leq 2$. Mark the points for $x = -2, -1, 0, 1, 2$.



Q17 The graph $y = k^x$, with $k > 0$, passes through $(3, 27)$. Find k .

Q18 The graphs are $y = 2^x$ (solid) and $y = x + 1$ (dashed). Read their intersection coordinates.

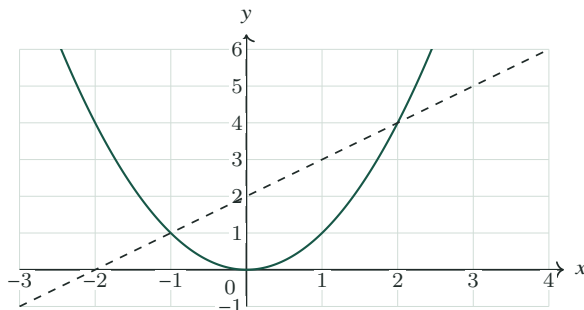


4. Intersections and graph reasoning

Example Problem 4

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Find the intersections of $y = x^2$ and $y = x + 2$.



The idea

1. Set the two expressions for y equal, solve for x , then calculate y .

Worked Solution

$$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$$

Rearrange.

$$(x - 2)(x + 1) = 0 \Rightarrow x = 2, -1$$

Factorise and solve.

$$(2, 4)$$

Substitute each x into either equation.

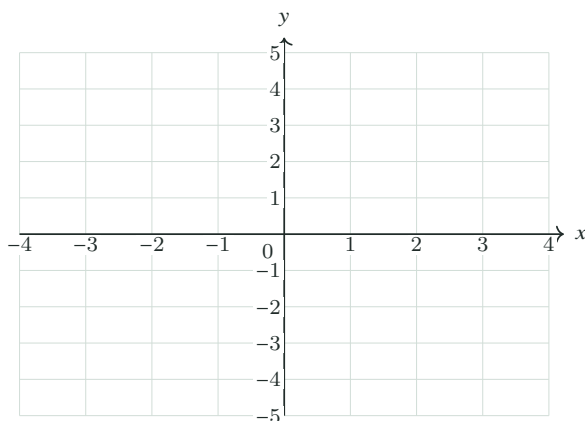
$$(-1, 1)$$

4 Intersections and graph reasoning

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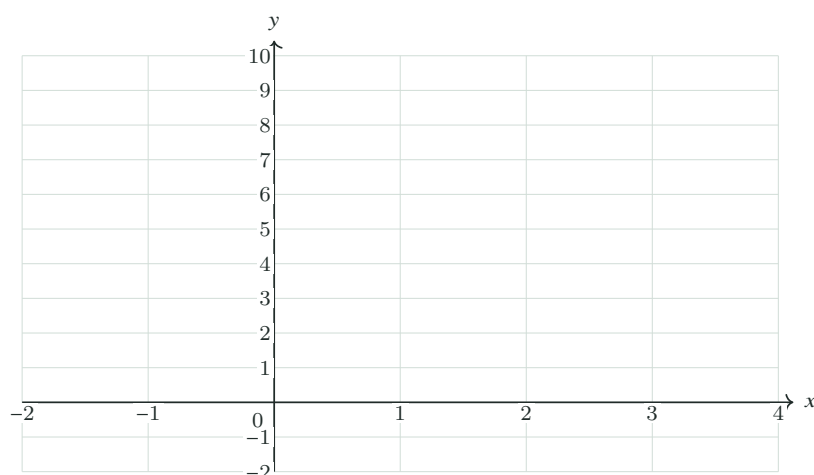
Q19 Sketch $y = -x^2 + 4$ on the grid. Label the roots and maximum point.

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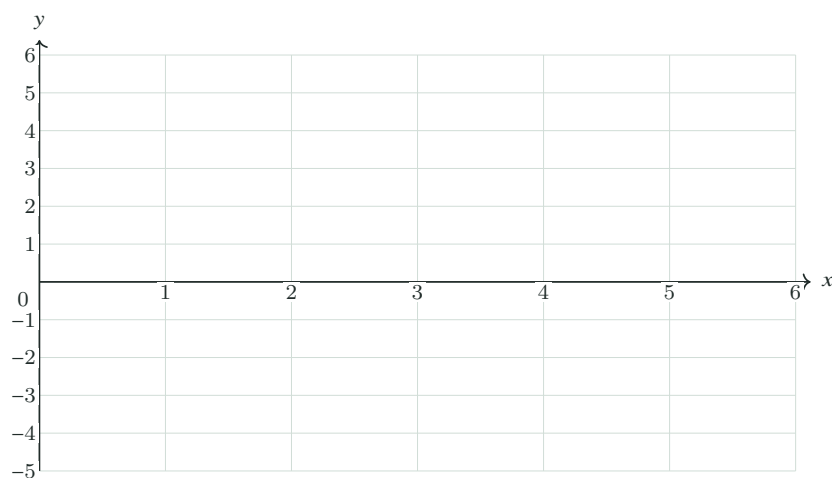
**Q20** Find the exact intersections of $y = x^2$ and $y = 2x + 3$.

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- Q21** Find the maximum value of $y = -2(x - 1)^2 + 8$, and the x -value where it occurs. Sketch the graph on the axes. ***



- Q22** A curve has equation $y = (x - a)^2 + b$. Its minimum point is $(3, -4)$. Write its equation. Sketch the graph on the axes. ***



- Q23** For which values of c does $y = x^2 + 2x + c$ have no x -intercepts? *****

Q24 Find the exact intersections of $y = 6/x$ and $y = x + 1$.

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Solving principles

1. Use key features

Mark roots, intercepts, turning points and asymptotes before sketching.

2. Keep the shape

Join points with a smooth curve when the equation defines a curved graph.

3. Check the scale

Read the labels on both axes before estimating.

4. Match both coordinates

Substitute an intersection into both equations.