

GCSE MATHS

Pythagoras and trigonometry

Worked Solutions

Q1

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$$AC^2 = 6^2 + 8^2 = 100$$

AC is the hypotenuse.

$$AC = 10 \text{ cm}$$

Take the positive square root.

Q2

★★

$$BC^2 = 17^2 - 15^2 = 64$$

Subtract the known shorter-side square from the hypotenuse square.

$$BC = 8 \text{ cm}$$

Take the square root.

Q3

★★

$$d^2 = 9^2 + 12^2 = 225$$

The diagonal is a right-triangle hypotenuse.

$$d = 15 \text{ cm}$$

Take the positive root.

Q4

★★

$$h^2 = 13^2 - 5^2 = 144$$

The perpendicular height bisects the base.

$$h = 12 \text{ cm}$$

Take the square root.

$$A = \frac{1}{2}(10)(12) = 60 \text{ cm}^2$$

Use half base times height.

Q5

★★

$$\text{change in } x = 6$$

Find horizontal and vertical separations.

$$\text{change in } y = 8$$

$$AB = \sqrt{6^2 + 8^2} = 10$$

Use Pythagoras on the coordinate triangle.

Q6

$$10^2 + 24^2 = 100 + 576 = 676$$

Add squares of the shorter sides.

$$26^2 = 676$$

Square the longest side.

$$10^2 + 24^2 = 26^2$$

The square of the longest side equals the sum of the other two squares, so the triangle is right-angled.

Q7

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$$\sin 35^\circ = \frac{BC}{12}$$

BC is opposite and AC is the hypotenuse.

$$BC = 12 \sin 35^\circ$$

Multiply by 12.

$$BC = 6.9 \text{ cm}$$

Evaluate in degrees and round.

Q8

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$$\cos 42^\circ = \frac{AB}{9}$$

AB is adjacent and AC is the hypotenuse.

$$AB = 9 \cos 42^\circ$$

Multiply by nine.

$$AB = 6.7 \text{ cm}$$

Evaluate and round.

Q9

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$$\tan 28^\circ = \frac{BC}{7}$$

Use opposite divided by adjacent.

$$BC = 7 \tan 28^\circ$$

Multiply by seven.

$$BC = 3.7 \text{ cm}$$

Evaluate and round.

Q10

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$$\sin A = \frac{5}{13}$$

Use opposite and hypotenuse.

$$A = \sin^{-1} \left(\frac{5}{13} \right)$$

Apply inverse sine.

$$A = 22.6^\circ$$

Evaluate in degrees.

Q11

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$$\cos A = \frac{8}{15}$$

$$A = \cos^{-1}\left(\frac{8}{15}\right)$$

$$A = 57.8^\circ$$

Use adjacent and hypotenuse.

Apply inverse cosine.

Evaluate and round.

Q12

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$$\tan A = \frac{11}{7}$$

$$A = \tan^{-1}\left(\frac{11}{7}\right)$$

$$A = 57.5^\circ$$

Use opposite and adjacent.

Apply inverse tangent.

Evaluate and round.

Q13

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$$\cos \theta = \frac{1.8}{5}$$

$$\theta = \cos^{-1}(0.36)$$

$$\theta = 68.9^\circ$$

The ground distance is adjacent; the ladder is the hypotenuse.

Use inverse cosine.

Evaluate in degrees.

Q14

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$$\tan 32^\circ = \frac{h}{35}$$

$$h = 35 \tan 32^\circ$$

$$h = 21.9 \text{ m}$$

Height is opposite and ground distance is adjacent.

Multiply by 35.

Evaluate and round.

Q15

$$\text{height difference} = 23 - 8 = 15 \text{ m}$$

$$\tan \theta = \frac{15}{24}$$

$$\theta = 32.0^\circ$$

Use the difference in height.

Form the triangle between the tops.

Apply inverse tangent and round.

Q16

$$\sin 52^\circ = \frac{9}{l}$$

$$l = \frac{9}{\sin 52^\circ}$$

$$l = 11.4 \text{ m}$$

The string is the hypotenuse.

Divide nine by the sine.

Evaluate without replacing the denominator by an early rounded value.

Q17

$$\frac{b}{2} = 10 \sin 20^\circ$$

$$h = 10 \cos 20^\circ$$

$$A = 100 \sin 20^\circ \cos 20^\circ$$

$$A = 32.1 \text{ cm}^2$$

The height bisects the vertex angle and base.

Find the height in the same right triangle.

Use half base times height.

Evaluate using unrounded lengths.

Q18

$$h = x \tan 50^\circ$$

$$h = (x + 20) \tan 30^\circ$$

$$x = \frac{20 \tan 30^\circ}{\tan 50^\circ - \tan 30^\circ}$$

$$h = 22.4 \text{ m}$$

Let x be the nearer distance.

Use the farther triangle.

Equate the heights and solve for x .

Multiply the unrounded x by $\tan 50^\circ$.

Q19

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$$4 \left(\frac{1}{2} \right) + 2 \left(\frac{1}{2} \right)$$

$$2 + 1 = 3$$

Use the exact values of $\sin 30^\circ$ and $\cos 60^\circ$.

Evaluate.

Q20

$$\tan 60^\circ = \frac{BC}{6}$$

$$BC = 6 \tan 60^\circ = 6\sqrt{3}$$

Use opposite and adjacent.

Use the exact tangent value.

Q21

(a) $d_{\text{base}} = \sqrt{3^2 + 4^2} = 5 \text{ cm}$

$$D = \sqrt{5^2 + 12^2} = 13 \text{ cm}$$

(b) $d_{\text{base}} = \sqrt{3^2 + 4^2} = 5 \text{ cm}$

$$\tan \theta = \frac{12}{5}$$

$$\theta = 67.4^\circ$$

Find the base diagonal first.

Use the vertical right triangle.

The diagonal AG lies above the base diagonal AC , so use triangle ACG .

Use the triangle with the vertical height.

Apply inverse tangent.

Q22

$$OC = \frac{\sqrt{8^2 + 8^2}}{2} = 4\sqrt{2}$$

$$\tan \theta = \frac{12}{4\sqrt{2}}$$

$$\theta = 64.8^\circ$$

Find the base-centre to corner distance.

The apex, centre and corner form a right triangle.

Apply inverse tangent and round.

Q23

$$OC = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$h^2 = 13^2 - (5\sqrt{2})^2 = 119$$

$$h = \sqrt{119} \text{ cm}$$

$$V = \frac{1}{3}(100)\sqrt{119}$$

Find half the base diagonal.

Use Pythagoras in the apex-centre-corner triangle.

Take the positive root.

Use one third times base area times vertical height.