

GCSE MATHS

Pythagoras and trigonometry

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Pythagoras

In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse, opposite the right angle.

Trigonometric ratios

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}.$$

Finding an angle

Use an inverse function: if $\tan \theta = \frac{3}{4}$, then $\theta = \tan^{-1}(\frac{3}{4})$. Use degree mode.

Working in three dimensions

Identify a right-angled triangle containing the required line or angle. A diagonal of the base may need to be found first.

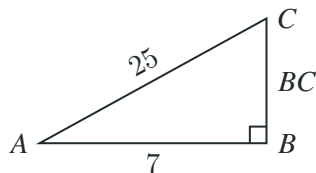
Use degree mode. Give non-exact lengths and angles to 1 decimal place unless the question asks otherwise. Diagrams are not drawn to scale.

1. Pythagoras in two dimensions

Example Problem 1

**

Triangle ABC is right-angled at B . $AC = 25$ cm and $AB = 7$ cm. Find BC .



The idea

1. The required side is shorter than the hypotenuse, so subtract the squares.
2. Take the square root after the subtraction.

Worked Solution

$$BC^2 + 7^2 = 25^2$$

Identify AC as the hypotenuse.

$$BC^2 = 625 - 49 = 576$$

Subtract the shorter-side square.

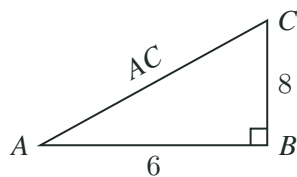
$$BC = 24 \text{ cm}$$

Take the positive square root.

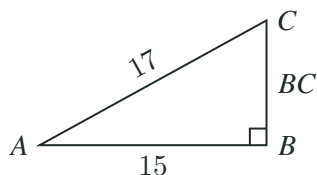
1 Pythagoras in two dimensions

Q1 Triangle ABC is right-angled at B . $AB = 6$ cm and $BC = 8$ cm. Find AC .

*

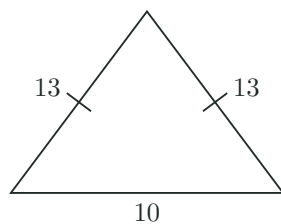
**Q2** Triangle ABC is right-angled at B . $AC = 17$ cm and $AB = 15$ cm. Find BC .

**

**Q3** A rectangle is 9 cm by 12 cm. Find its diagonal.

**

Q4 An isosceles triangle has equal sides 13 cm and base 10 cm. Find its area. **



Q5 Find the distance between $A(-2, 3)$ and $B(4, 11)$. **

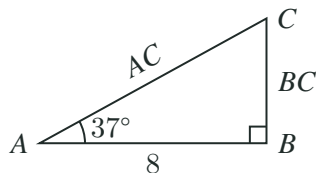
Q6 A triangle has sides 10 cm, 24 cm and 26 cm. Prove that it is right-angled. ***

2. Sine, cosine and tangent

Example Problem 2

**

Triangle ABC is right-angled at B . $AB = 8$ cm and angle BAC is 37° . Find BC to 1 decimal place.



The idea

1. Label the sides relative to angle A .
2. Use tangent because the hypotenuse is not involved.

Worked Solution

$$\tan 37^\circ = \frac{BC}{8}$$

$$BC = 8 \tan 37^\circ$$

$$BC = 6.0 \text{ cm}$$

The known and required sides are adjacent and opposite.

Multiply both sides by eight.

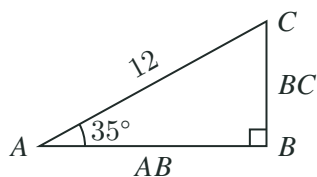
Evaluate in degree mode and round.

2 Sine, cosine and tangent

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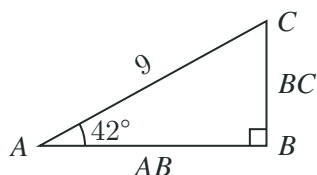
- Q7** Triangle ABC is right-angled at B . $AC = 12$ cm and angle BAC is 35° . Find BC to 1 decimal place.

**



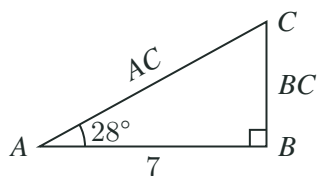
- Q8** Triangle ABC is right-angled at B . $AC = 9$ cm and angle BAC is 42° . Find AB to 1 decimal place.

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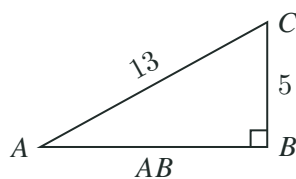


- Q9** Triangle ABC is right-angled at B . $AB = 7$ cm and angle BAC is 28° . Find BC to 1 decimal place.

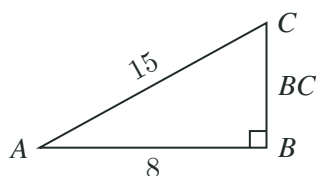
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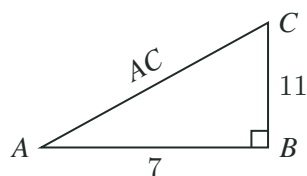
- Q10** Triangle ABC is right-angled at B . $BC = 5$ cm and $AC = 13$ cm. Find angle BAC to 1 decimal place. **



- Q11** Triangle ABC is right-angled at B . $AB = 8$ cm and $AC = 15$ cm. Find angle BAC to 1 decimal place. **



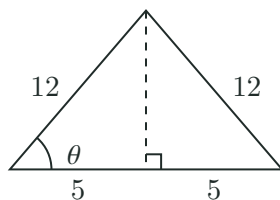
- Q12** Triangle ABC is right-angled at B . $AB = 7$ cm and $BC = 11$ cm. Find angle BAC to 1 decimal place. **



3. Connected triangles

Example Problem 3

An isosceles triangle has equal sides 12 cm and base 10 cm. Find its base angle to 1 decimal place.



The idea

1. Split the isosceles triangle into two congruent right triangles.
2. Use half the base as the adjacent side.

Worked Solution

$$\frac{b}{2} = 5 \text{ cm}$$

$$\cos \theta = \frac{5}{12}$$

$$\theta = 65.4^\circ$$

The perpendicular height bisects the base.

Use the half-triangle with hypotenuse 12.

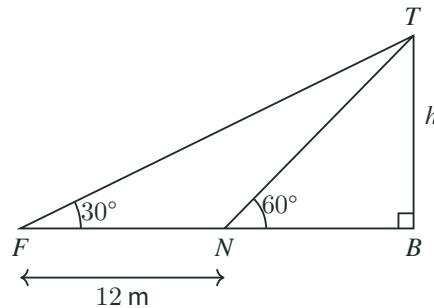
Apply inverse cosine.

3. Connected triangles

Example Problem 4

★★★★

Two points F and N lie on level ground on the same side of a vertical tower BT . N is 12 m nearer the tower. The angles of elevation are 60° at N and 30° at F . Find the tower height to 1 decimal place.



The idea

1. Let x be the nearer horizontal distance NB . The farther distance FB is $x + 12$.
2. Write the height in two ways, equate them, then solve for x .

Worked Solution

$$h = x \tan 60^\circ$$

Use the nearer right-angled triangle.

$$h = (x + 12) \tan 30^\circ$$

Use the farther triangle; its height is the same.

$$x(\tan 60^\circ - \tan 30^\circ) = 12 \tan 30^\circ$$

Equate, expand and collect the x terms.

$$x = 6\text{ m}$$

Divide by the difference of the tangents.

$$h = 6 \tan 60^\circ = 10.4\text{ m}$$

Substitute the nearer distance and round.

3 Connected triangles

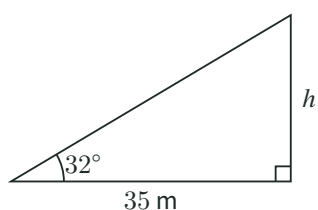
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Q13 A 5 m ladder rests against a vertical wall with its foot 1.8 m from the wall on horizontal ground. Find its angle to the ground to 1 decimal place.

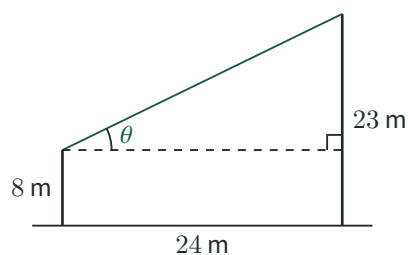
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Q14 From a point on horizontal ground 35 m from a tower's base, the angle of elevation to its top is 32° . Find the tower's height to 1 decimal place.

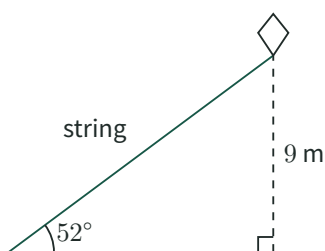
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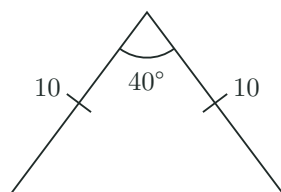
- Q15** Two vertical poles stand on horizontal ground 24 m apart. Their heights are 8 m and 23 m. Find the angle of elevation from the top of the shorter pole to the top of the taller pole, to 1 decimal place.



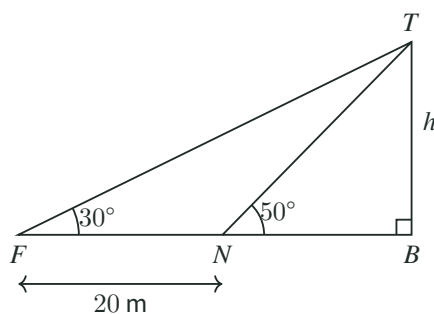
- Q16** A kite is 9 m vertically above a point on level ground. Its straight string makes an angle of 52° with the horizontal at ground level. Find the string length to 1 decimal place.



- Q17** An isosceles triangle has equal sides 10 cm and vertex angle 40° . Find its area to 1 decimal place.



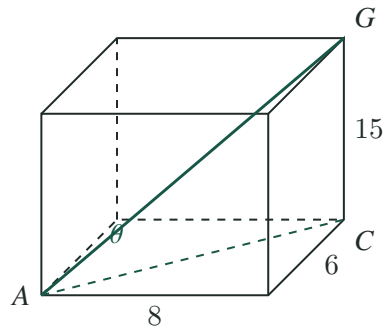
- Q18** Two points lie on horizontal ground on the same side of a tower, in line with its base. The nearer point is 20 m closer. From the nearer point, the angle of elevation of the top is 50° ; from the farther point it is 30° . Find the tower height to 1 decimal place.



4. Exact values and three dimensions

Example Problem 5

A cuboid has horizontal base 6 cm by 8 cm and height 15 cm. Find the angle between its longest diagonal AG and its base, to 1 decimal place.



The idea

1. Find the base diagonal before the angle.
2. The angle to the base uses that diagonal as its adjacent side.

Worked Solution

$$d_{\text{base}} = \sqrt{6^2 + 8^2} = 10 \text{ cm}$$

$$\tan \theta = \frac{15}{10}$$

$$\theta = 56.3^\circ$$

Find the diagonal of the base first.

Use the vertical right triangle.

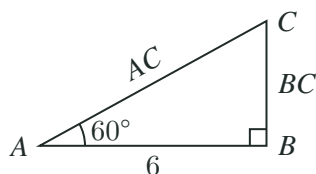
Apply inverse tangent in degrees.

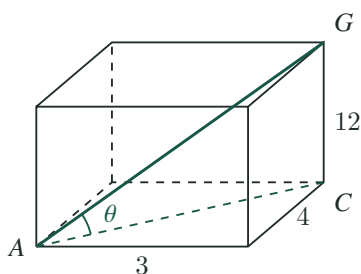
4 Exact values and three dimensions

Q19 Evaluate $4 \sin 30^\circ + 2 \cos 60^\circ$ exactly.

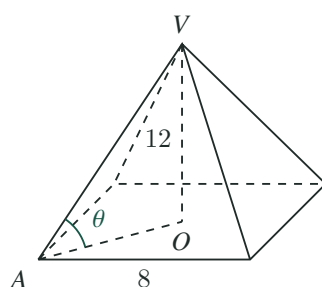
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Q20 Triangle ABC is right-angled at B . $AB = 6$ cm and angle BAC is 60° . Find the exact length BC .

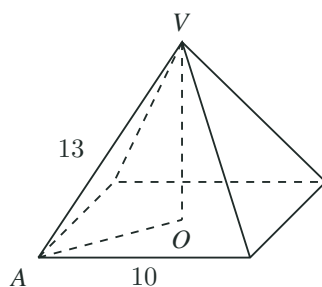
**Q21** A cuboid has a horizontal base 3 cm by 4 cm and vertical height 12 cm.

(a) Find the length of its longest diagonal, AG .**(b)** Find the angle θ between AG and the base, to 1 decimal place.

- Q22** A square-based pyramid has base side 8 cm. Its apex V is 12 cm vertically above the base centre O . Find the angle θ between edge AV and the base, to 1 decimal place.



- Q23** A square-based pyramid has base side 10 cm. Each sloping edge from apex V to a base corner is 13 cm. V is vertically above the centre O . Find the exact volume.



Solving principles

1. Mark the hypotenuse

It is opposite the right angle and is the longest side. Subtract squares to find a shorter side.

2. Choose the ratio from the sides

Label opposite and adjacent relative to the angle you are using.

3. Carry an unrounded value

In connected triangles, use the full first result in the next calculation.

4. Draw the right-angled triangle

Draw a perpendicular from the elevated point to the horizontal base. Mark the given angle and the lengths you know.