

GCSE MATHS

Probability trees

Worked Solutions

Q1

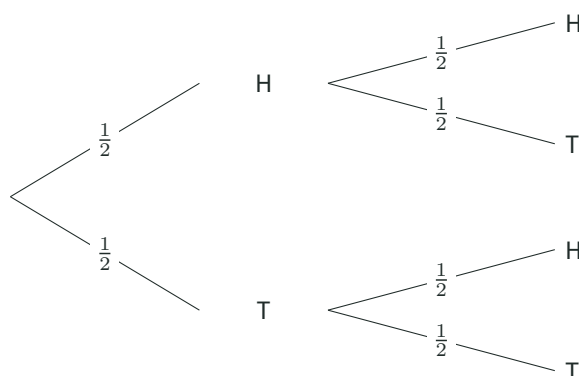
★

$$P(HH) = \frac{1}{2} \times \frac{1}{2}$$

$$P(HH) = \frac{1}{4}$$

Multiply the independent head probabilities.

Simplify.



Q2

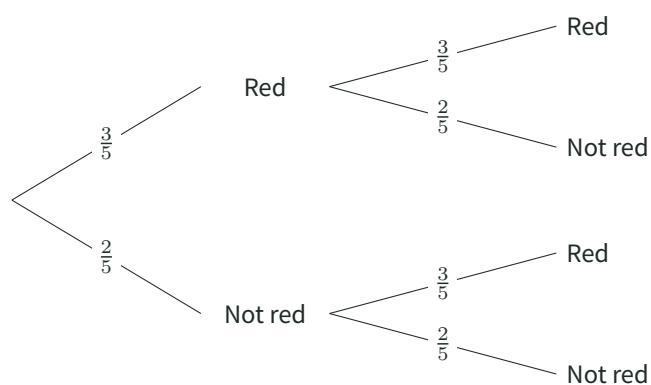
★★

$$P(\text{not red}) = \frac{2}{5}$$

$$P(RN) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$$

Subtract the red probability from one.

The order is specified, so use one route.



Q3

**

$$P(SM) = 0.7(0.3) = 0.21$$

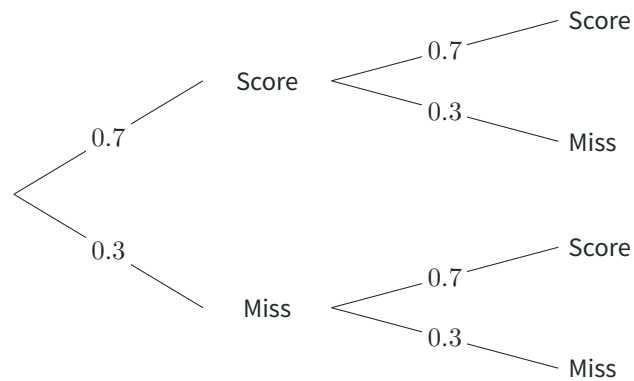
Score then miss.

$$P(MS) = 0.3(0.7) = 0.21$$

Miss then score.

$$P(\text{exactly one}) = 0.42$$

Add the probabilities of the two separate routes.



Q4

**

$$P(\text{no six}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

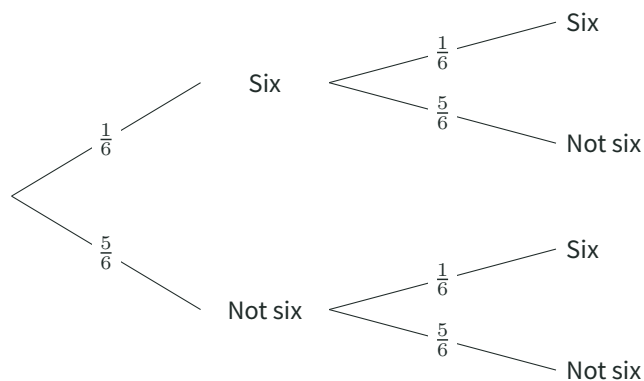
Use the complement event.

$$P(\text{at least one six}) = 1 - \frac{25}{36}$$

Subtract from one.

$$\frac{11}{36}$$

Simplify.



Q5

**

$$0.9(0.2) = 0.18$$

First passes and second fails.

$$0.1(0.8) = 0.08$$

First fails and second passes.

$$0.18 + 0.08 = 0.26$$

Add the routes.

Q6

$$P(HHT) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3}$$

$$P(HHT) = \frac{4}{27}$$

$$P(\text{exactly two}) = 3 \times \frac{4}{27} = \frac{4}{9}$$

Find one route with two heads.

Multiply the branch probabilities.

HHT, *HTH* and *THH* have the same probability.

Q7

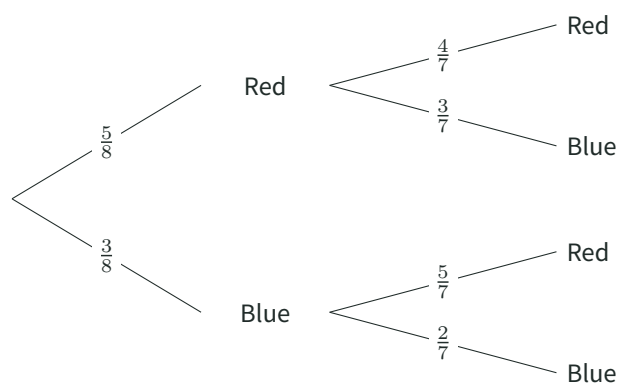
**

$$P(RR) = \frac{5}{8} \times \frac{4}{7}$$

$$P(RR) = \frac{5}{14}$$

After a red, four red counters remain among seven.

Multiply and simplify.



Q8

**

$$P(RB) = \frac{4}{10} \times \frac{6}{9}$$

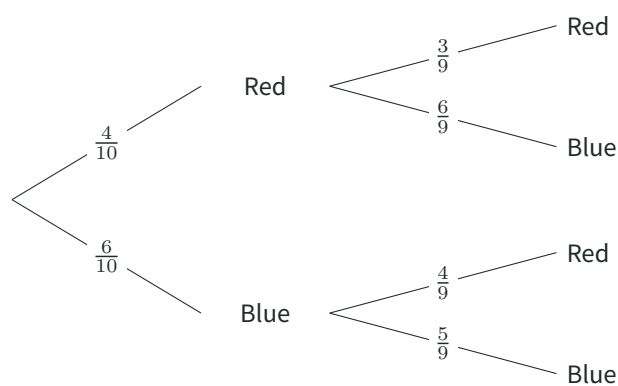
$$P(BR) = \frac{6}{10} \times \frac{4}{9}$$

$$P(\text{one each}) = \frac{48}{90} = \frac{8}{15}$$

Find red then blue.

Find blue then red.

Add both orders.



Q9

**

$$P(WW) = \frac{9}{12} \times \frac{8}{11} = \frac{6}{11}$$

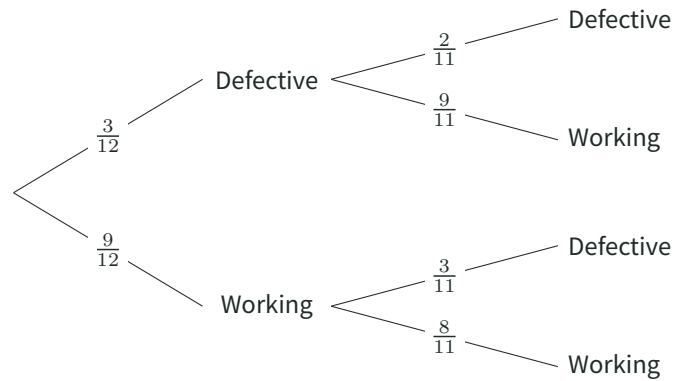
$$P(\text{at least one defective}) = 1 - \frac{6}{11}$$

$$\frac{5}{11}$$

Calculate the complement: neither is defective.

Subtract from one.

Simplify.



Q10

$$P(BBB) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$$

$$P(BBB) = \frac{120}{720} = \frac{1}{6}$$

Reduce both the black count and total each time.

Multiply and simplify.

Q11

$$P(RRB) = \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{1}{5}$$

$$P(RBR) = P(BRR) = \frac{1}{5}$$

$$P(\text{exactly two red}) = \frac{3}{5}$$

Find one order.

The same counts give the same product in each order.

Add all three orders.

Q12

$$P_{\text{with}} = \frac{5}{10} \times \frac{5}{10} = \frac{1}{4}$$

$$P_{\text{without}} = \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$$

$$\frac{1}{4} - \frac{2}{9} = \frac{1}{36}$$

Replacement restores the counts.

Without replacement, only four red remain.

Subtract using a common denominator.

Q13

(a) $P(R \text{ and } L) = 0.3 \times 0.4$

$$P(R \text{ and } L) = 0.12$$

(b) $P(RL) = 0.3(0.4) = 0.12$

$$P(NL) = 0.7(0.1) = 0.07$$

$$P(L) = 0.12 + 0.07 = 0.19$$

(c) $P(RL) = 0.3(0.4) = 0.12$

$$P(L) = 0.12 + 0.7(0.1) = 0.19$$

$$\text{Required probability} = \frac{0.12}{0.19} = \frac{12}{19}$$

Follow the rain and late route.

Multiply.

Find the rainy late route.

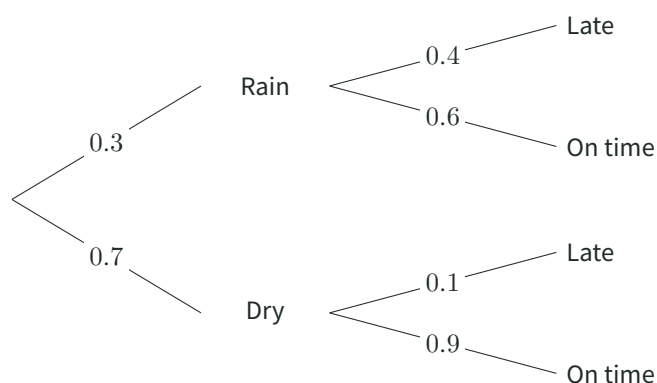
Find the dry late route.

Add both ways to be late.

Find the probability that both events happen.

Find the total probability of being late.

Restrict to the late journeys.



Q14

$$P(AR) = \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

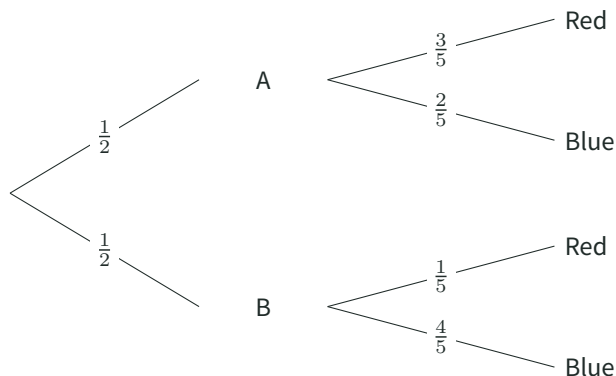
$$P(R) = \frac{3}{10} + \frac{1}{2} \times \frac{1}{5} = \frac{4}{10}$$

$$\text{Required probability} = \frac{3/10}{4/10} = \frac{3}{4}$$

Find the A and red route.

Add red routes from both boxes.

Divide joint probability by total red probability.



Q15

$$P(HH) = \frac{1}{4}$$

$$P(\text{at least one H}) = \frac{3}{4}$$

$$\frac{1/4}{3/4} = \frac{1}{3}$$

Find the event required.

Only TT is excluded.

Restrict to the three equally likely remaining outcomes.

Q16

$$P(RR) = \frac{4}{6} \times \frac{3}{5} = \frac{2}{5}$$

$$P(BB) = \frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$$

$$P(\text{same}) = \frac{7}{15}$$

$$\text{Required probability} = \frac{2/5}{7/15} = \frac{6}{7}$$

Find two red.

Find two blue.

Add the same-colour routes.

Divide by the restricted total.

Q17

$$p^2 = 0.49$$

$$p = 0.7$$

Multiply the equal independent probabilities.

Take the non-negative square root.

Q18

$$(1 - p)^2 = 0.16$$

$$1 - p = 0.4$$

$$p = 0.6$$

The probability of no successes is $1 - 0.84$.

Use the non-negative square root.

Subtract from one.

Q19

$$0.2(1 - p) + 0.8p = 0.38$$

$$0.2 + 0.6p = 0.38$$

$$p = 0.3$$

Add the two routes with exactly one failure.

Expand and collect p terms.

Subtract 0.2 and divide by 0.6.

Q20

★★★★

$$\frac{3}{n+3} \times \frac{2}{n+2} = \frac{1}{5}$$

$$(n+3)(n+2) = 30$$

$$n^2 + 5n - 24 = 0$$

$$(n+8)(n-3) = 0$$

$$n = 3$$

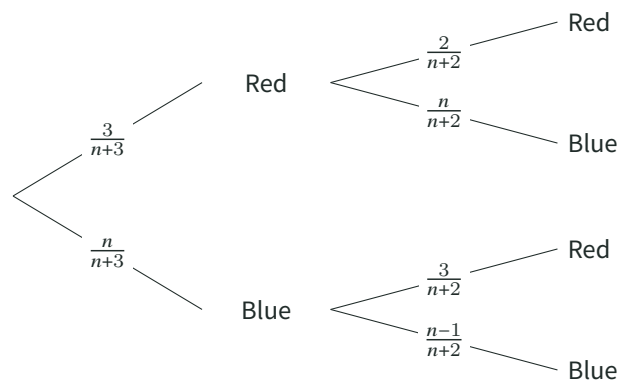
Update the red count and total after the first draw.

Cross-multiply.

Expand and rearrange.

Factorise.

Reject -8 because it is not a count.



Q21

★★★★

$$\frac{8n}{(n+4)(n+3)} = \frac{8}{15}$$

$$15n = (n+4)(n+3)$$

$$n^2 - 8n + 12 = 0$$

$$(n-2)(n-6) = 0$$

$$n = 2 \text{ or } 6$$

Add red-blue and blue-red.

Cross-multiply and cancel eight.

Expand and rearrange.

Factorise.

Both positive whole-number counts are possible.

Q22

★★★★

$$P(B \text{ transferred}, R \text{ drawn}) = \frac{3}{5} \times \frac{4}{6} = \frac{2}{5}$$

$$P(R \text{ transferred}, R \text{ drawn}) = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$P(R \text{ drawn}) = \frac{2}{5} + \frac{1}{3} = \frac{11}{15}$$

$$\text{Required probability} = \frac{2/5}{11/15} = \frac{6}{11}$$

After a blue transfer, four of the six counters in B are red.

After a red transfer, five of the six are red.

Add the two transfer routes.

Restrict to draws that are red.

