

GCSE MATHS

Probability trees

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Independent events	Q1–6	***	3
2. Without replacement	Q7–12	***	7
3. Conditional probability	Q13–16	***	11
4. Unknown probabilities and counts	Q17–22	*****	14
Solving principles			19

Topic recap

Branches

The probabilities leaving any one point add to one. Label the second stage using the information on its first branch.

Multiply along a route

For a sequence of events, multiply the branch probabilities on that route.

Add separate routes

For exactly one success in two attempts, add success–failure and failure–success. These routes cannot happen together.

Given information

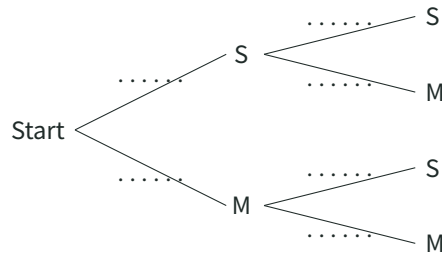
Given information narrows the outcomes being considered. Divide the probability of the required route by the total probability of all routes satisfying that information.

1. Independent events

Example Problem 1

**

Complete the tree. A player scores with probability $\frac{3}{4}$ on each of two independent shots. Find the probability of exactly one score.



The idea

1. Draw two stages with score and miss branches. Independence keeps the second-stage probabilities unchanged.
2. Multiply along each required route, then add the two results.

Worked Solution

$$P(SM) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

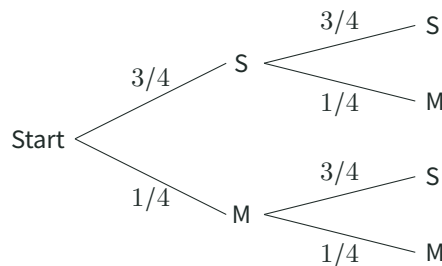
Score then miss.

$$P(MS) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

Miss then score.

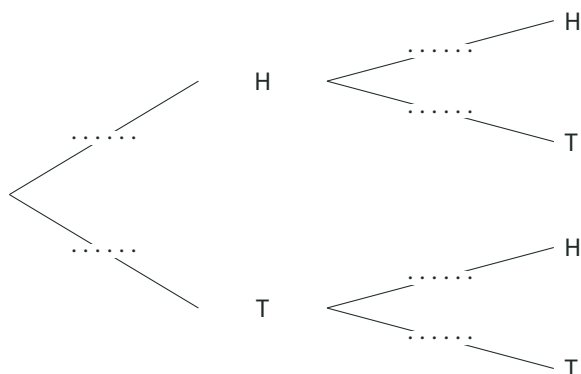
$$P(\text{exactly one}) = \frac{3}{8}$$

Add the probabilities of both separate routes.

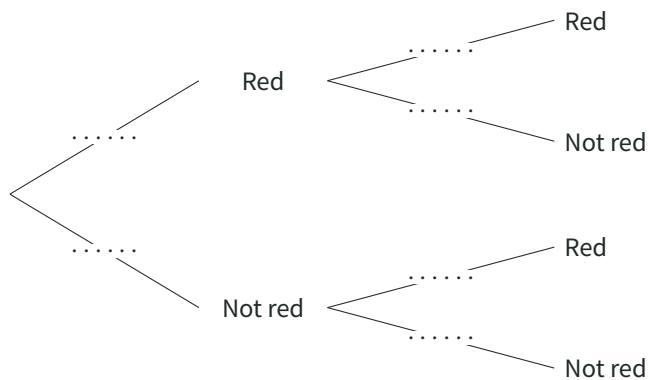


1 Independent events

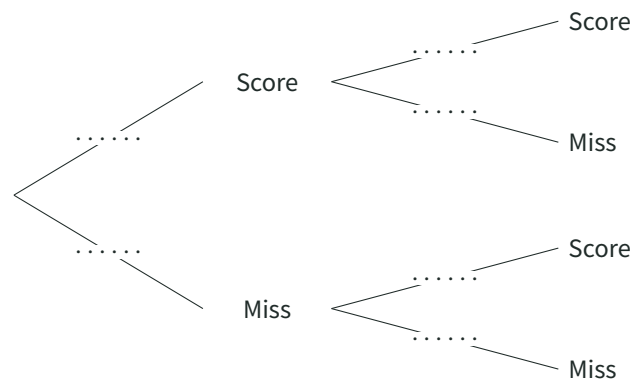
Q1 A fair coin is tossed twice. Complete the probabilities on the tree. Then find the probability of two heads. *



Q2 A spinner lands on red with probability $\frac{3}{5}$. It is spun twice independently. Complete the probabilities on the tree. Then find the probability of red then not red. **



- Q3** A player scores on each independent shot with probability 0.7. Complete the probabilities on the tree. Then find the probability of exactly one score in two shots. **



- Q4** A fair six-sided die is rolled twice independently. Draw and label a probability tree. Then find the probability of at least one six. **

- Q5** Two independent machines pass a test with probabilities 0.9 and 0.8. Find the probability that exactly one passes. **

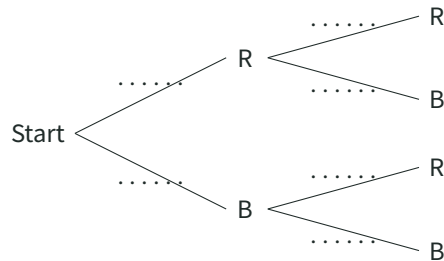
Q6 A biased coin has probability $\frac{2}{3}$ of heads. It is tossed independently three times. Find the probability of exactly two heads.

2. Without replacement

Example Problem 2

**

Complete the tree. A bag contains 5 red and 2 blue counters. Two are drawn without replacement. Find the probability of one of each colour.



The idea

1. Reduce the second-stage total to six.
2. Include both red–blue and blue–red.

Worked Solution

$$P(RB) = \frac{5}{7} \times \frac{2}{6} = \frac{5}{21}$$

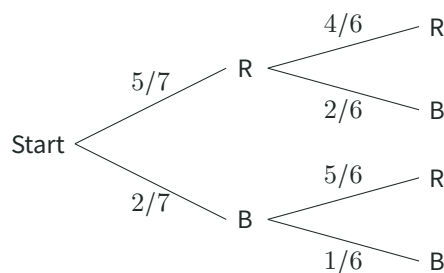
$$P(BR) = \frac{2}{7} \times \frac{5}{6} = \frac{5}{21}$$

$$P(\text{one each}) = \frac{10}{21}$$

After red, there are six counters and still two blue.

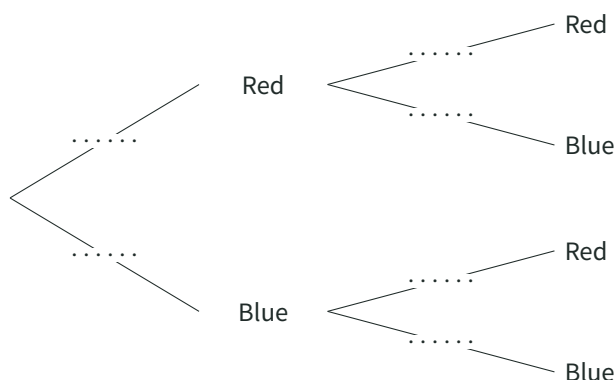
After blue, there are six counters and still five red.

Add both orders.



2 Without replacement

- Q7** A bag contains 5 red and 3 blue counters. Two are drawn without replacement. Complete the probabilities on the tree. Then find the probability both are red. **

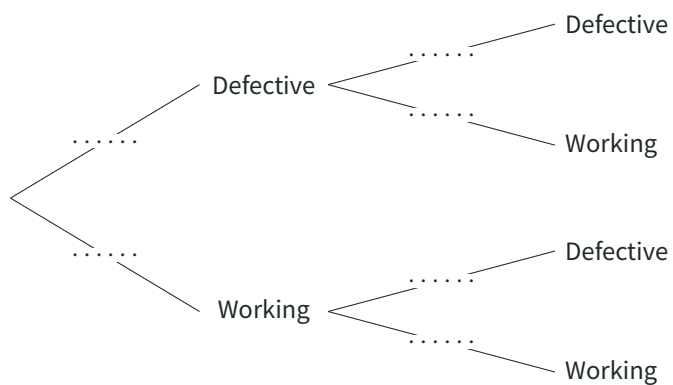


- Q8** A bag contains 4 red and 6 blue counters. Two are drawn without replacement. Draw and label a probability tree. Then find the probability of one of each colour. **

**

- Q9** A box contains 3 defective and 9 working bulbs. Two are selected without replacement. Complete the probabilities on the tree. Then find the probability that at least one is defective.

**



Q10 A bag contains 6 black and 4 white counters. Three are drawn without replacement. Find the probability all three are black.

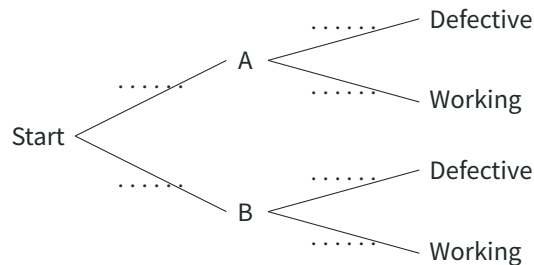
Q11 A bag contains 3 red and 2 blue counters. Three are drawn without replacement. Find the probability of exactly two red counters.

Q12 A bag contains 5 red and 5 blue counters. Two are drawn. Find how much greater the probability of two red counters is with replacement than without replacement.

3. Conditional probability

Example Problem 3

Complete the tree. A factory uses machine A for 60% of its items. A produces 2% defective items; B produces 5%. Given that an item is defective, find the probability it came from A.



The idea

1. For each machine, find the probability that an item comes from it and is faulty.
2. Divide the A-and-defective probability by the total defective probability.

Worked Solution

$$P(AD) = 0.6(0.02) = 0.012$$

Find the A and defective route.

$$P(BD) = 0.4(0.05) = 0.020$$

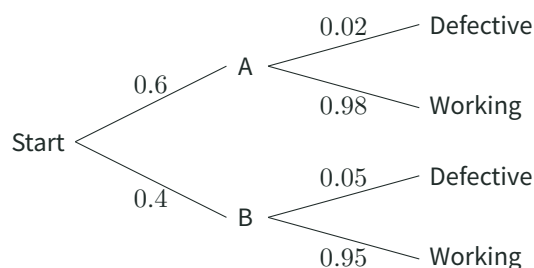
Find the B and defective route.

$$P(D) = 0.032$$

Add all defective routes.

$$\text{Required probability} = \frac{0.012}{0.032} = \frac{3}{8}$$

Use only defective items as the new total.



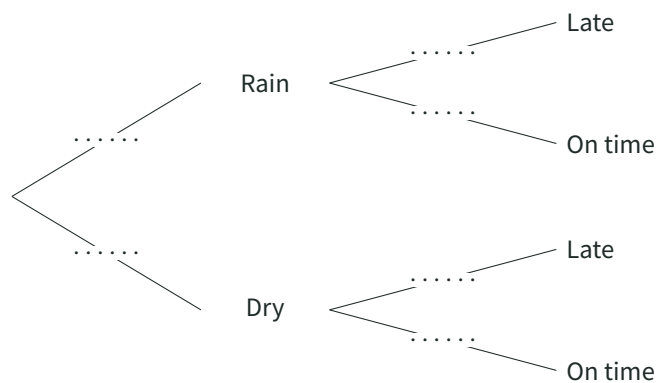
3 Conditional probability

Q13 The probability of rain is 0.3. The probability that a bus is late is 0.4 if it rains and 0.1 otherwise.

(a) Complete the tree. Find the probability that it rains and the bus is late.

(b) Find the probability that the bus is late.

(c) Given that the bus is late, find the probability that it rained.



- Q14** Box A contains 3 red and 2 blue counters. Box B contains 1 red and 4 blue counters. A box is chosen with equal probability, then one counter is drawn. Draw and label a probability tree. Given that it is red, find the probability it came from A.

- Q15** A fair coin is tossed twice. Given that at least one toss is heads, find the probability that both are heads.

- Q16** A bag contains 4 red and 2 blue counters. Two are drawn without replacement. Given that both are the same colour, find the probability both are red.

4. Unknown probabilities and counts

Example Problem 4

A bag contains 4 red and n blue counters. Two are drawn without replacement. The probability both are red is $\frac{2}{7}$. Find n .

The idea

1. Put the unknown total on both stages, reducing it by one on stage two.
2. Clear the fractions, solve the quadratic and check the count.

Worked Solution

$$\frac{4}{n+4} \times \frac{3}{n+3} = \frac{2}{7}$$

$$(n+4)(n+3) = 42$$

$$n^2 + 7n - 30 = 0$$

$$(n+10)(n-3) = 0$$

$$n = 3$$

Write the red-red route.

Cross-multiply and simplify.

Expand and rearrange.

Factorise.

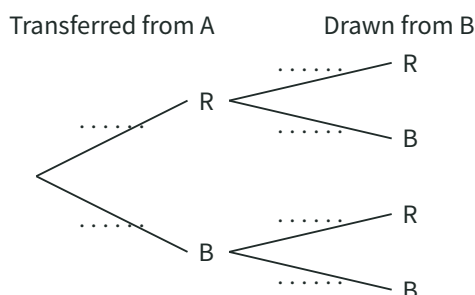
Reject the negative count.

4. Unknown probabilities and counts

Example Problem 5

★★★★

Complete the tree. Bag A contains 3 red and 1 blue counter. Bag B contains 2 red and 2 blue counters. One counter is transferred at random from *A* to *B*, then one is drawn from *B*. Given that the drawn counter is red, find the probability that the transferred counter was blue.



The idea

1. The transferred colour changes the counts in *B*. There are five counters in *B* after either transfer.
2. Keep the two routes ending in red; find the blue-transfer share of their total.

Worked Solution

$$P(\text{blue, then red}) = \frac{1}{4} \times \frac{2}{5} = \frac{1}{10}$$

After a blue transfer, *B* has two red and three blue.

$$P(\text{red, then red}) = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$$

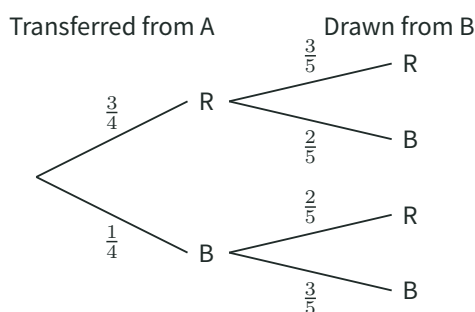
After a red transfer, *B* has three red and two blue.

$$P(\text{red drawn}) = \frac{1}{10} + \frac{9}{20} = \frac{11}{20}$$

Add both routes ending in red.

$$\text{Required probability} = \frac{1/10}{11/20} = \frac{2}{11}$$

Divide the blue-then-red route by the red-drawn total.



4 Unknown probabilities and counts

★★★★

Q17 A player succeeds with probability p on each of two independent attempts. The probability of two successes is 0.49. Find p .

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Q18 Two independent attempts have the same success probability p . The probability of at least one success is 0.84. Find p .

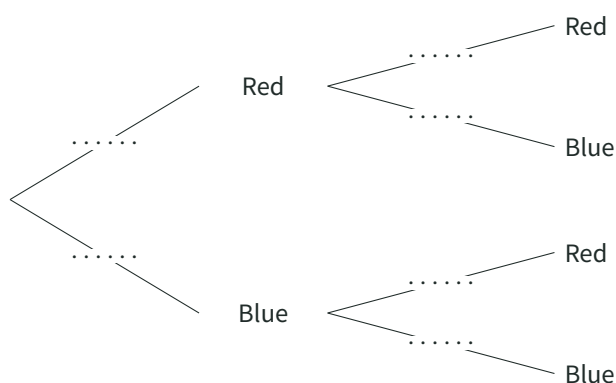
★★★

Q19 Two independent machines fail with probabilities 0.2 and p . The probability that exactly one fails is 0.38. Find p .

★★★

- Q20** A bag contains 3 red counters and n blue counters. Two are drawn without replacement. The probability that both are red is $\frac{1}{5}$. Complete the probabilities on the tree. Then find n .

★★★★



- Q21** A bag contains n red and 4 blue counters. Two are drawn without replacement. The probability of one of each colour is $\frac{8}{15}$. Find all possible values of n .

★★★★

Q22 Bag A contains 2 red and 3 blue counters. Bag B contains 4 red and 1 blue counter. One counter is moved at random from A to B , then one is drawn from B . Draw and label a probability tree. Given that the drawn counter is red, find the probability that the transferred counter was blue.

Solving principles

1. Check replacement

Replacing an object restores the original counts. Without replacement, both the total and one category count change.

2. List the routes

Write all orders that satisfy the event. Two colours can occur in either order.

3. Use the complement

At least one success is one minus no successes.

4. Check the permitted values

Probabilities lie between zero and one. Numbers of objects must be whole numbers.