

GCSE MATHS

Probability and sample spaces

Worked Solutions

Q1

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$$\{3, 6\}$$

List the successful outcomes.

$$P = 2/6 = 1/3$$

Divide by six equally likely results.

Q2

★

$$N = 5 + 3 + 2 = 10$$

Count all counters.

$$P(B) = 3/10$$

Three counters are blue.

Q3

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$$P(A') = 1 - 0.37 = 0.63$$

Use the complement.

Q4

★★

$$p = 1 - 0.2 - 0.35 = 0.45$$

The three outcome probabilities sum to one.

Q5

★★

$$2, 3, 5, 7, 11, 13, 17, 19$$

List the primes; 1 is not prime.

$$P = 8/20 = 2/5$$

Count successes and simplify.

Q6

★★

(a) $P(6) = \frac{1}{6}$

One of the six equally likely outcomes is a six.

(b) $P(\text{not } 6) = \frac{5}{6}$

The other five outcomes are not sixes.

Q7

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(a) HH, HT, TH, TT

Keep the first and second coin distinct.

(b) $P(\text{one H}) = 2/4 = 1/2$ HT and TH are the successful outcomes.

Q8

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 $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)$

List the ordered pairs with sum 7.

$$P = 6/36 = 1/6$$

There are 36 equally likely pairs.

Q9

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 $(1, 1), (2, 2), \dots, (6, 6)$

The diagonal pairs match.

$$P = 6/36 = 1/6$$

Count six matching pairs.

Q10

 $(1, 8), (2, 6), (2, 8), (3, 6), (3, 8)$

List the successful ordered pairs.

$$P = 5/12$$

There are twelve equally likely pairs.

Q11

$$6 + 6 - 1 = 11$$

Count pairs with a 5 first or second; subtract the double-counted $(5, 5)$.

$$P = 11/36$$

Divide by all ordered pairs.

Q12

$$4^2 - 3^2 = 7$$

Count pairs with both results at most 4, then remove pairs with both at most 3.

$$P = 7/36$$

The remaining pairs have maximum 4.

Q13

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$$p = 84/150 = 0.56$$

Divide heads by total tosses.

Q14

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$$p = 9/600 = 0.015$$

Estimate the faulty proportion.

$$E = 4000(0.015) = 60$$

Scale to 4000 items.

Q15

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$$E = 250(0.18) = 45$$

Multiply the number of trials by the probability.

Q16

$$p = (18 + 72)/(50 + 150) = 90/200 = 0.45$$

Combine counts, rather than averaging the two proportions equally.

Q17

$$P(6) = \frac{1}{6}$$

Each roll is independent of the earlier rolls.

Q18

$$0.07n = 10.5$$

Write expected frequency as np.

$$n = 10.5/0.07 = 150$$

Divide by the win probability.

Q19

$$12/(12 + b) = 3/7$$

Use the total including the unknown blues.

$$84 = 36 + 3b \Rightarrow b = 16$$

Cross-multiply and solve.

Q20

$$(4 + x)/(10 + x) = 3/5$$

Add x to the red count and total.

$$20 + 5x = 30 + 3x \Rightarrow x = 5$$

Solve.

Q21

$$R = 2k$$

Represent the original counts.

$$B = 3k$$

$$2k + 4 = 3k$$

Half red means equal red and blue counts.

$$k = 4 \Rightarrow N = 5k = 20$$

Find the original total.

Q22

$$(n - 4)/n = 3/5$$

There are $n - 4$ successful sectors.

$$5n - 20 = 3n \Rightarrow n = 10$$

Cross-multiply and solve.

Q23

$$3, 5, 7, 11$$

The possible sums are 3, 5, 7 and 11. Each uses one odd and one even result; sum 2 has odd product.

$$2 + 4 + 6 + 2 = 14$$

Count the ordered pairs for those sums.

$$P = 14/36 = 7/18$$

Divide by all outcomes.

Q24

$$P(G) = 1 - 2/5 - 1/3 = 4/15$$

Find the remaining probability.

$$8/N = 4/15 \Rightarrow N = 30$$

Use the green count to find the total.