

GCSE MATHS

Probability and sample spaces

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Single events and complements	Q1–6	**	3
2. Sample spaces	Q7–12	***	5
3. Relative frequency and expectation	Q13–18	***	8
4. Probability with constraints	Q19–24	****	10
Solving principles			13

Topic recap

Equally likely outcomes

$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$. Count only equally likely outcomes this way.

Complements

$P(\text{not } A) = 1 - P(A)$. The probabilities of all possible, mutually exclusive outcomes sum to 1.

Sample spaces

List outcomes systematically, using a table when there are two choices. Ordered pairs distinguish a first result from a second.

Relative frequency

Estimate probability using successes/trials. Expected frequency is np for n trials with success probability p ; it need not be a whole number.

1. Single events and complements

Example Problem 1

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A bag has 6 black, 4 white and 5 yellow counters. Find the probability a randomly chosen counter is not white.

The idea

1. An event and its complement cover every possible outcome.

Worked Solution

$$N = 6 + 4 + 5 = 15$$

Find the total.

$$P(\text{not } W) = 1 - 4/15 = 11/15$$

Subtract the white probability from one.

1 Single events and complements

**

Q1 A fair six-sided die is rolled. Find the probability of a multiple of 3.

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Q2 A bag has 5 red, 3 blue and 2 green counters. One is selected at random. Find the probability it is blue.

*

Q3 $P(A) = 0.37$. Find $P(A')$, the probability A does not happen.

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Q4 A spinner has outcomes A , B and C with probabilities 0.2, p and 0.35. Find p .

**

Q5 A card is chosen at random from cards numbered 1 to 20. Find the probability its number is prime.

**

Q6 A fair six-sided die is rolled. Find the probability of (a) a six; (b) a number other than six.

**

2. Sample spaces

Example Problem 2

Two independent spinners have equal sectors labelled 1, 2, 3 and 1, 2, 3, 4. Find the probability the sum is 5.

Sum	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7

The idea

1. Make one row for each result on the first spinner, and one column for each on the second.

Worked Solution

$$(1, 4), (2, 3), (3, 2)$$

$$N = 3 \times 4 = 12$$

$$P = 3/12 = 1/4$$

List the ordered pairs with sum 5.

Count the complete sample space.

Divide successful pairs by all pairs.

2 Sample spaces

Q7 Two fair coins are tossed. (a) List the sample space. (b) Find the probability of exactly one head.

**

Q8 Two fair six-sided dice are rolled. Find the probability that their sum is 7.

**

Q9 Two fair dice are rolled. Find the probability that both show the same number.

**

- Q10** Spinner A has equal sectors numbered 1, 2, 3. Spinner B has equal sectors numbered 2, 4, 6, 8. They are spun independently. Find the probability their sum exceeds 7. ***

- Q11** Two fair dice are rolled. Find the probability their product is a multiple of 5. ***

- Q12** Two fair dice are rolled. Find the probability the highest score shown is 4. ***

3. Relative frequency and expectation

Example Problem 3

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A sample has 12 late buses out of 80. Estimate the number late out of 300 under the same conditions.

The idea

1. Use relative frequency as the probability estimate.

Worked Solution

$$p = 12/80 = 0.15$$

Use the observed proportion.

$$E = 300(0.15) = 45$$

Multiply by the future number of buses.

3 Relative frequency and expectation

Q13 A coin gives 84 heads in 150 tosses. Use relative frequency to estimate its probability of heads.

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Q14 A factory sample has 9 faulty items out of 600. Estimate the number faulty in 4000 items made under the same conditions.

**

Q15 A biased die has probability 0.18 of showing a six. Find the expected number of sixes in 250 rolls.

**

Q16 Two experiments under the same conditions give 18 successes in 50 trials and 72 in 150. Combine them to estimate the probability of success.

Q17 A fair six-sided die has been rolled 20 times without a six. Find the probability of a six on the next roll.

Q18 Sam plays a game with a win probability of 0.07. Over n games, the expected number of wins is 10.5. Find n .

4. Probability with constraints

Example Problem 4

A bag has 3 red and 7 blue counters. How many red counters must be added to make the probability of red $1/2$?

The idea

1. Write the new counts before forming the probability equation.

Worked Solution

$$(3 + x)/(10 + x) = 1/2$$

Both the successful count and total increase.

$$6 + 2x = 10 + x$$

Cross-multiply.

$$x = 4$$

Subtract x and 6.

4 Probability with constraints

★★★★

Q19 A bag contains only red and blue counters. There are 12 red counters and the probability of red is $\frac{3}{7}$. How many blue counters are there?

★★★

Q20 A bag has 4 red and 6 blue counters. How many red counters must be added to make the probability of red $\frac{3}{5}$?

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Q21 A bag contains red and blue counters in ratio 2:3. Adding 4 red counters makes the probability of red $\frac{1}{2}$. Find the original total.

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Q22 A fair spinner has equal sectors numbered 1 to n . The probability of a number greater than 4 is $\frac{3}{5}$. Find n .

Q23 Two fair dice are rolled. Find the probability that their sum is prime and their product is even.

Q24 A bag contains counters that are all red, blue or green. The probabilities of red and blue are $\frac{2}{5}$ and $\frac{1}{3}$. There are 8 green counters. Find the total number of counters.

Solving principles

1. Define the outcome

Decide whether order matters and whether outcomes have equal chances.

2. Use the complement

Counting outcomes outside an event can be shorter than counting the event itself.

3. Interpret an expected number

Multiply the probability by the number of trials. The actual number of successes can differ from this prediction.

4. Use whole-number constraints

Numbers of objects must be non-negative integers, even when a probability equation has other solutions.