

GCSE MATHS

Linear equations and modelling

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Balance

An equation states that two expressions are equal. Apply the same operation to both sides.

Brackets

Expand each bracket completely, then collect terms before isolating the unknown.

Fractions

Multiply every term on both sides by a common denominator to clear fractions.

Model

Define the unknown with its units. Translate each relationship into an expression before forming the equation.

1. Balancing linear equations

Example Problem 1

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Solve $7x + 3 = 4x + 18$.

The idea

1. Move variable terms to one side and constants to the other.

Worked Solution

$$3x + 3 = 18$$

Subtract $4x$ from both sides.

$$3x = 15$$

Subtract 3.

$$x = 5$$

Divide by 3.

1 Balancing linear equations

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Q1 Solve $3x + 7 = 25$.

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Q2 Solve $5x - 9 = 2x + 12$.

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Q3 Solve $18 - 4x = 6$.

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Q4 Solve $7x + 4 = 3x - 16$.

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Q5 Solve $\frac{x}{3} + 5 = 9$.

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Q6 Solve $\frac{3x}{2} - 4 = 8$.

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2. Brackets and fractional terms

Example Problem 2

Solve $\frac{x+1}{3} + \frac{x}{2} = 7$.

The idea

1. Use the lowest common denominator to remove both fractions in one step.

Worked Solution

$$2(x + 1) + 3x = 42$$

Multiply every term by 6.

$$5x + 2 = 42$$

Expand and collect.

$$x = 8$$

Subtract 2 and divide by 5.

2 Brackets and fractional terms

Q7 Solve $3(x - 4) = 2(x + 1)$.

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Q8 Solve $5 - 2(x + 3) = 3x + 4$.

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Q9 Solve $\frac{x+2}{3} = 5$.

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Q10 Solve $\frac{2x-1}{3} = \frac{x+4}{2}$.

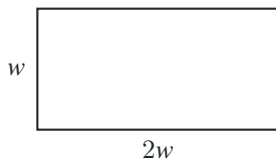
Q11 Solve $\frac{x}{4} + \frac{x-1}{3} = 5$.

Q12 Solve $\frac{2}{5}(x-3) = \frac{3}{4}$.

3. Forming equations

Example Problem 3

A rectangle is twice as long as it is wide. Its perimeter is 36 cm. Find its width.



The idea

1. Write each length using one unknown, then use the perimeter.

Worked Solution

$$2(w + 2w) = 36$$

$$6w = 36 \Rightarrow w = 6$$

Let w be the width in centimetres.

Solve the equation.

3 Forming equations

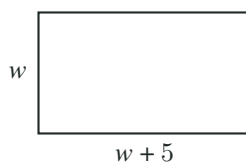
Q13 Three consecutive integers have sum 72. Find them.

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Q14 A taxi charges £4 plus £2.50 per kilometre. A journey costs £29. Find its length.

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Q15 A rectangle has length 5 cm more than its width and perimeter 42 cm. Find its dimensions.



Q16 A mother is three times as old as her daughter. In 12 years, the mother will be twice as old as her daughter. Find their current ages.

Q17 A class buys 4 identical notebooks and a £3 pen for £17. Find the cost of each notebook.

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Q18 A 60 cm wire is cut into two pieces. One is 8 cm shorter than three times the other. Find both lengths.

4. Equations with no solution or many solutions

Example Problem 4

Solve $2(x + 3) = 2x + 8$, or show that it has no solution.

The idea

1. If all variable terms cancel, inspect the remaining statement.

Worked Solution

$$2x + 6 = 2x + 8$$

Expand the brackets.

$$6 = 8$$

Subtract $2x$ from both sides.

No solution

The equality is false for every value of x .

4 Equations with no solution or many solutions

Q19 Plan A costs £15 plus £3 per visit. Plan B costs £39 plus £1 per visit. For how many visits do they cost the same?

Q20 Solve $4(2x - 3) = 8x + 1$, or explain why there is no solution.

Q21 Solve $3(2x + 4) = 6(x + 2)$.

Q22 Adult tickets cost £9 and child tickets £5. A group of 14 pays £94. How many adult tickets were bought?

Q23 A coach trip costs £240 for the coach, plus £6 per pupil. The pupils share the total cost equally. Four more pupils join, bringing the cost per pupil down to £16. Find the original number of pupils.

Q24 A fraction has numerator 3 less than its denominator. Adding 2 to both numerator and denominator gives $\frac{3}{4}$. Find the original fraction.

Solving principles

1. Keep equality

Each step must have the same solutions as the previous equation.

2. Use all terms

When clearing fractions, multiply terms without fractions too.

3. Check in the context

An algebraic result must satisfy any whole-number, positive-length or capacity restriction.

4. Substitute back

Test the answer in the original equation, not only the last rearranged line.