

GCSE MATHS

Iteration

Worked Solutions

Q1

**

$$x_1 = (2 + 6)/2 = 4.0000$$

Substitute the starting value, 2.

$$x_2 = (4 + 6)/2 = 5.0000$$

Use 4 as the next input.

$$x_3 = (5 + 6)/2 = 5.5000$$

Use 5 as the next input.

Q2

**

$$x_1 = \sqrt{10 + 3} \approx 3.6056$$

Substitute 3 and retain the full calculator value.

$$x_2 = \sqrt{10 + x_1} \approx 3.6886$$

Use the stored value of x_1 .

$$x_3 = \sqrt{10 + x_2} \approx 3.6998$$

Use the stored value of x_2 ; report 4 d.p.

Q3

**

$$x_1 = 1 + 6/2 = 4.0000$$

Substitute the starting value, 2.

$$x_2 = 1 + 6/4 = 2.5000$$

Use 4 as the next input.

$$x_3 = 1 + 6/2.5 = 3.4000$$

Use 2.5 as the next input.

Q4

**

$$x_2 = (13 + 3)/2 = 8$$

Reverse the operation: add 3, then divide by 2.

$$x_1 = (8 + 3)/2 = 5.5$$

Repeat backwards.

$$x_0 = (5.5 + 3)/2 = 4.25$$

Repeat once more.

Q5

**

$$7 = 2 + c \Rightarrow c = 5$$

Substitute the first two values.

$$x_2 = \frac{1}{2}(7) + 5 = 8.5$$

Use the same constant.

Q6

**

$$x_1 = \sqrt{10}$$

Calculate the first new value.

$$x_2 = \sqrt{\sqrt{10} + 8} = 3.3410 \dots$$

Substitute x_1 , not the starting value.

Q7

$$x^2 = x + 12$$

Add $x + 12$.

$$x = \sqrt{x + 12}$$

Take the positive square root.

Q8

$$x^3 = 7 - x$$

Isolate the cube.

$$x = \sqrt[3]{7 - x}$$

Take the cube root.

Q9

$$x(x^2 + 2) = 10$$

Factor out x .

$$x = \frac{10}{x^2 + 2}$$

Divide by the positive denominator.

Q10

$$2x^2 = x^2 + 5$$

$$x = \frac{1}{2}(x + 5/x)$$

Add x^2 to each side.

Divide by $2x$, using x non-zero.

Q11

$$L = \sqrt{6 + L}$$

$$L^2 - L - 6 = 0$$

$$(L - 3)(L + 2) = 0$$

$$L = 3$$

Replace both successive values by the limit.

Square and rearrange.

Factorise.

The square root cannot give the negative value -2 .

Q12

(a) $x_1 = 6, x_2 = 2, x_3 = 6, x_4 = 2$

(b) $2 \mapsto 6 \mapsto 2$

Divide 12 by the previous term.

The pair repeats forever, so there is no single limiting value.

Q13

**

$$f(1) = -3$$

$$f(2) = 5$$

$$1 < r < 2$$

Evaluate the lower endpoint.

Evaluate the upper endpoint.

The graph has no breaks and changes sign, so it crosses the x -axis between the two values tested.

Q14

**

$$f(1.5) = -3.625$$

$$f(2.5) = 6.625$$

$$1.5 < r < 2.5$$

Evaluate the lower endpoint.

Evaluate the upper endpoint.

The graph has no breaks and changes sign, so it crosses the x -axis between these values.

Q15

$$f(1.51) = -0.047049$$

$$f(1.52) = 0.031808$$

$$1.51 < r < 1.52$$

$$r = 1.5 \quad (1 \text{ d.p.})$$

Substitute at both endpoints.

The graph has no breaks and changes sign, so it crosses the x -axis between these values.

Every value in that interval rounds to 1.5.

Q16

$$2.345 < r < 2.355$$

$$r = 2.35 \quad (2 \text{ d.p.})$$

The unique root lies between the sign-changing endpoints.

These are the rounding boundaries for 2.35.

Q17

$$1.2344 \rightarrow 1.23$$

$$1.2356 \rightarrow 1.24$$

Choose two possible values on opposite sides of the rounding boundary.

Q18

$$x_1 \approx 1.7321$$

$$x_2 \approx 1.8077$$

$$x_3 \approx 1.7867$$

$$x_4 \approx 1.7926$$

Substitute the previous unrounded value.

Substitute the previous unrounded value.

Substitute the previous unrounded value.

Substitute the previous unrounded value.

Q19

$$x_1 = (6 + 9/6)/2 = 3.75$$

$$x_2 = (3.75 + 9/3.75)/2 = 3.075$$

$$x_3 = (3.075 + 9/3.075)/2 \approx 3.0009$$

Substitute 6.

Use 3.75.

Round to 4 decimal places.

Q20

$$A : 2, 1, 2$$

$$B : \frac{3}{2}, \frac{17}{12}, \frac{577}{408}$$

$$1.5, 1.4167, 1.4142$$

A repeats its starting pair.

Apply the averaging formula successively.

B gives values approaching the positive square root.

Q21

$$L = 3L - 4$$

$$2L = 4 \Rightarrow L = 2$$

An unchanged value must reproduce itself.

Solve.

Q22

$$x_1 \approx 2.666667$$

$$x_2 \approx 2.645833$$

$$x_3 \approx 2.645751$$

$$x_4 \approx 2.645751$$

$$x \approx 2.646 \quad (3 \text{ d.p.})$$

Use the initial value $x_0 = 3$.

Use the previous unrounded value.

The displayed values settle to the required accuracy.

Round the numerical solution.

Q23

(a) $a + b = 3$

$$3a + b = 4$$

$$2a = 1 \Rightarrow a = \frac{1}{2}$$

$$b = \frac{5}{2}$$

Use both transitions.

Subtract and solve.

(b) $L = \frac{1}{2}L + \frac{5}{2} \Rightarrow L = 5$

Solve the equation for the limit.

Q24

$$f(1.3025) = -0.00099375$$

Test the lower rounding boundary.

$$f(1.3035) = 0.00261225$$

Test the upper rounding boundary.

$$1.3025 < r < 1.3035$$

The quadratic changes sign, so its graph crosses the x -axis between these values.

$$r = 1.303 \quad (3 \text{ d.p.})$$

Both boundaries surround the required rounded value.