

Inequality regions

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Boundary lines

Replace the inequality by equality to draw its boundary. Use a solid line for an included boundary and a dashed line for a strict inequality.

Choosing a side

Test a point off the boundary. Shade the side satisfying the inequality. In this pack shade the required region, not the excluded region.

Common regions

A solution must satisfy every inequality. Keep only the overlap of the required regions.

Quadratic signs

The x -values where the graph is above the x -axis make its expression positive. Below the axis makes it negative. Include roots only for non-strict inequalities.

1. Boundaries and regions

Example Problem 1

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Show the region $y > 2$ on a coordinate grid.

The idea

1. Draw a dashed horizontal boundary, then choose points above it.

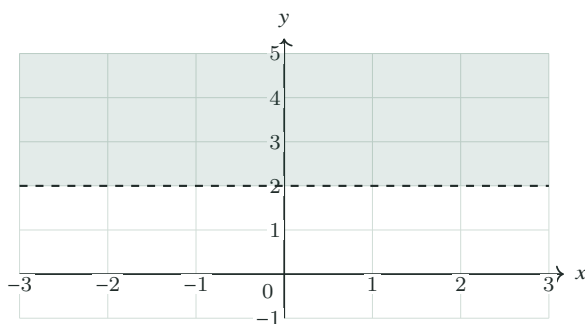
Worked Solution

$$y = 2$$

The boundary is excluded, so draw it dashed.

$$y > 2$$

Shade the region above the line.



1 Boundaries and regions

Q1 Does $(2, 5)$ satisfy $y > 2x$? Explain.

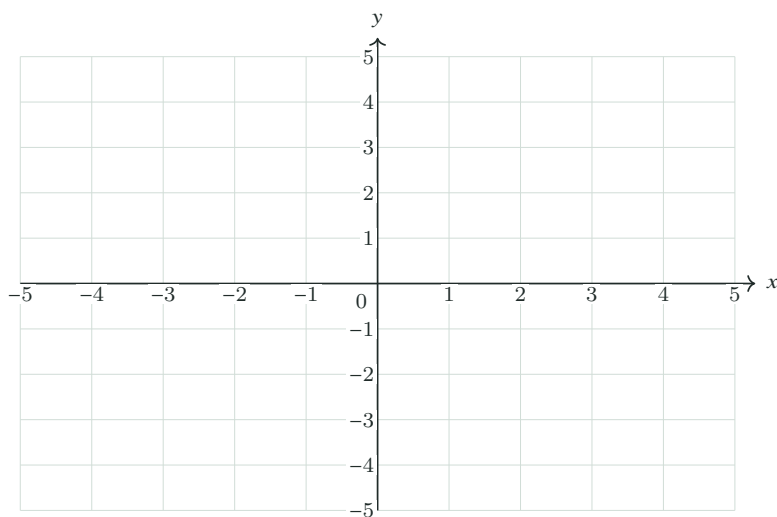
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Q2 Does $(-1, 2)$ satisfy $x + y \leq 0$? Explain.

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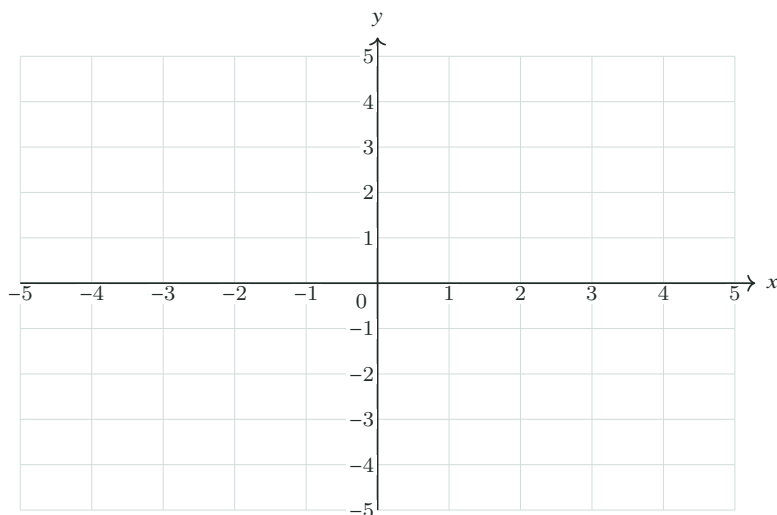
Q3 Draw $x = -2$ and shade the region $x \geq -2$.

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Q4 Draw the boundary and shade $y < 3$.

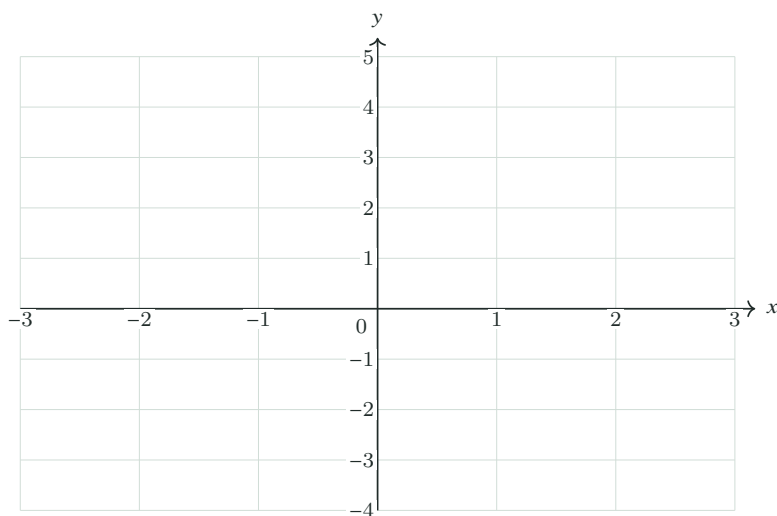
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Q5 Give two intercepts of the boundary line for $x + 2y \leq 6$.

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Q6 Draw the boundary and shade $y \geq 2x - 1$.



2. Combining linear inequalities

Example Problem 2

Describe the region satisfying $x \geq 0$, $y \geq 0$ and $x + y \leq 4$.

The idea

1. Use the first quadrant, then keep the side below the diagonal line.

Worked Solution

$$x + y = 4 \Rightarrow (0, 4), (4, 0)$$

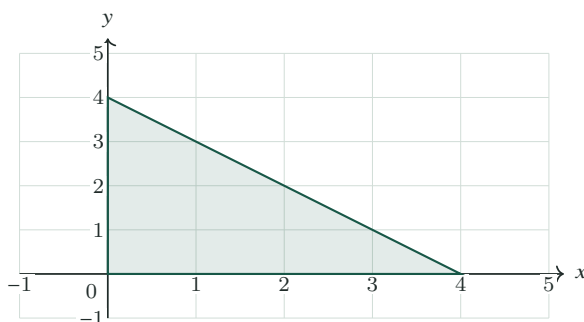
Find two boundary intercepts.

$$(0, 0) : 0 + 0 \leq 4$$

The origin is on the required side.

Vertices $(0, 0)$, $(4, 0)$, $(0, 4)$

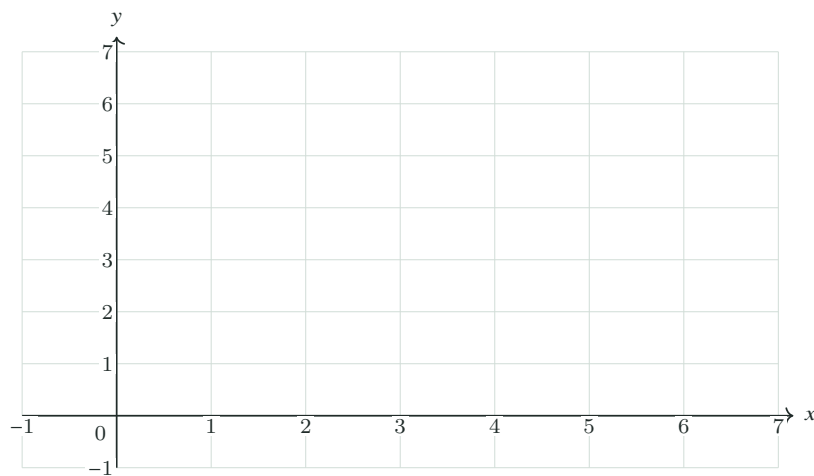
All three boundaries are included.

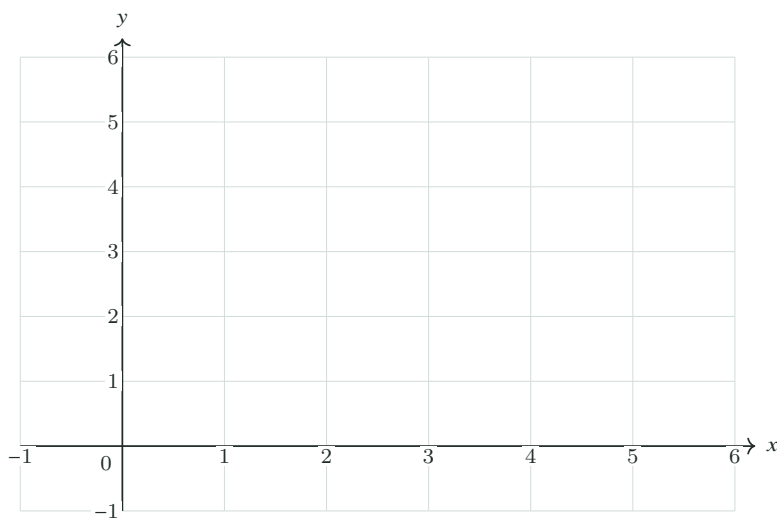


2 Combining linear inequalities

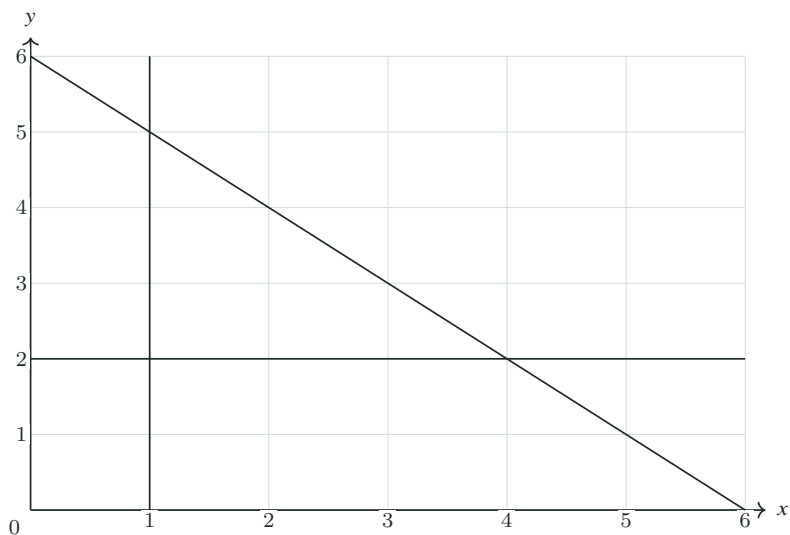
Q7 Draw the boundaries and shade the region $1 \leq x < 4$ on the axes.

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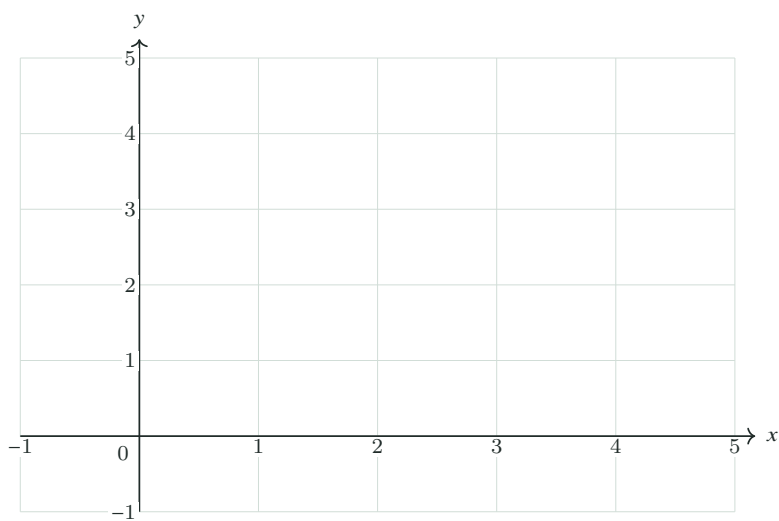
**Q8** Shade $x \geq 0$, $y \geq 0$ and $x + y \leq 5$. State its vertices.



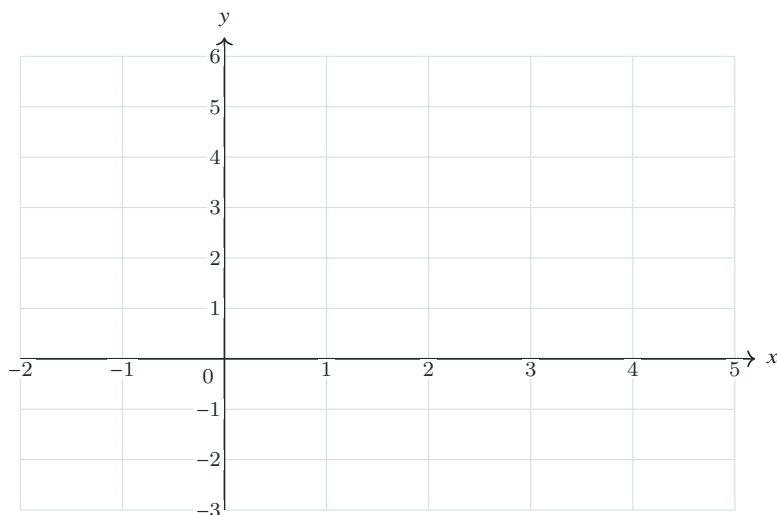
Q9 Find the vertices of $x \geq 1$, $y \geq 2$, $x + y \leq 6$.



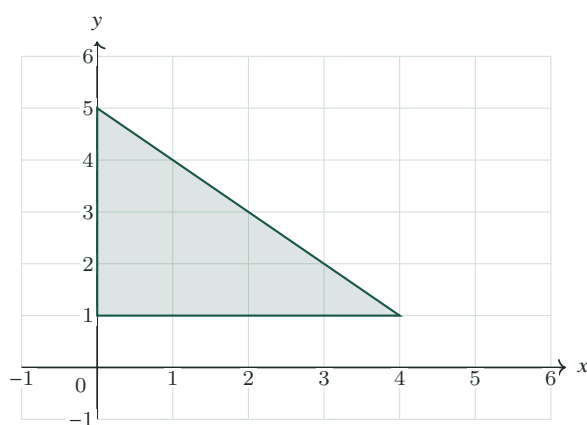
Q10 Shade $y \geq x$, $y \leq 4$ and $x \geq 0$. State the vertices.



Q11 Shade $y > x - 1$ and $y \leq -x + 3$. Is $(2, 1)$ included?



Q12 Write three inequalities for the shaded triangle. All boundaries are included.



3. Integer points and constraints

Example Problem 3

Count points with integer coordinates satisfying $x \geq 1$, $y \geq 1$ and $x + y \leq 5$.

The idea

1. For each possible x , count the positive integer y -values.

Worked Solution

$$x = 1 : 4; \quad x = 2 : 3; \quad x = 3 : 2; \quad x = 4 : 1$$

The largest y is $5 - x$.

$$4 + 3 + 2 + 1 = 10$$

Add the counts.

3. Integer points and constraints

Example Problem 4

Integers x and y satisfy $x \geq 0$, $y \geq 0$, $x + y \leq 5$ and $y \leq x$. For each possible y -value, find the greatest allowed x -value. Hence find the greatest value of $x + 2y$.

The idea

1. Find the possible whole-number values of y . For each, use the largest allowed x and compare the totals.

Worked Solution

$$y \leq x \leq 5 - y \Rightarrow y \leq 2.5$$

So y can only be 0, 1 or 2.

$$y = 0 : \quad x = 5$$

For $y = 0$, the greatest allowed x is 5.

$$x + 2y = 5$$

$$y = 1 : \quad x = 4$$

Repeat with $y = 1$.

$$x + 2y = 6$$

$$y = 2 : \quad x = 3$$

Repeat with $y = 2$.

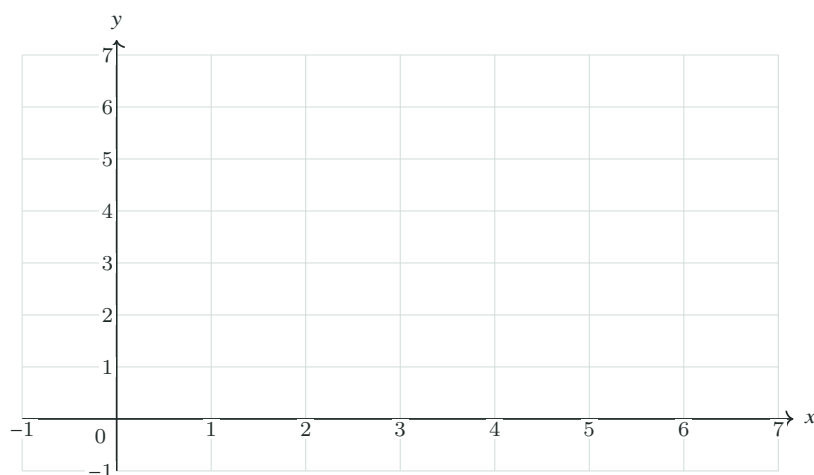
$$x + 2y = 7$$

Greatest value 7 at (3, 2)

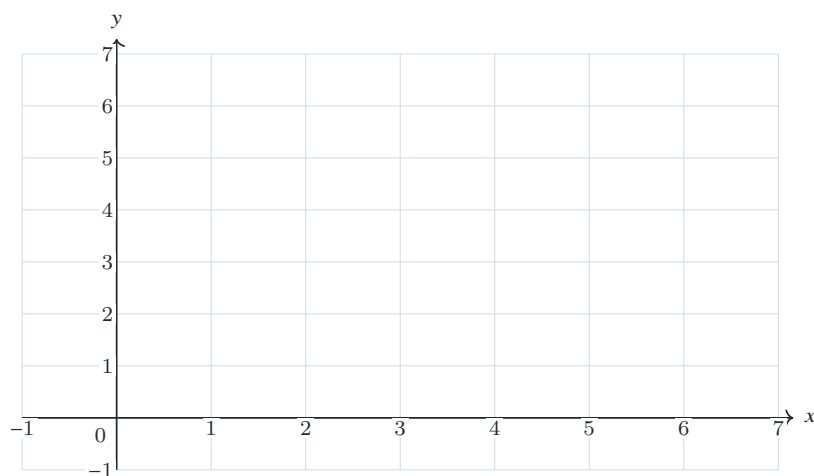
Compare the three totals.

3 Integer points and constraints

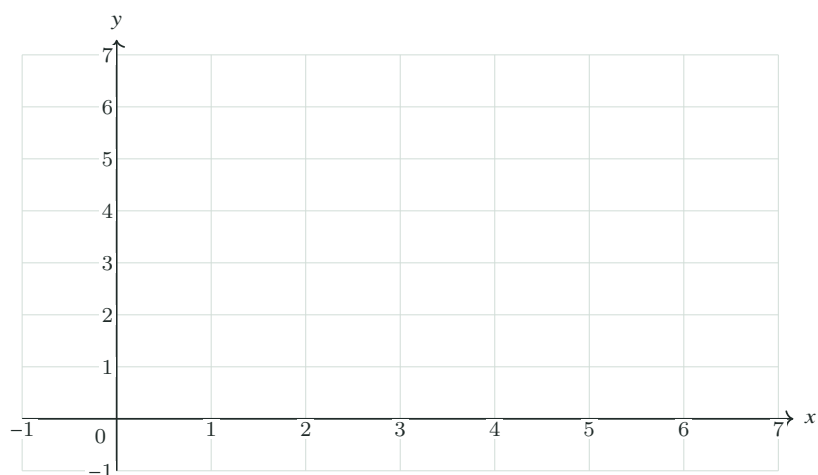
Q13 x and y are integers. Find how many points (x, y) satisfy $0 \leq x \leq 3$ and $1 \leq y \leq 4$. Mark these points on the grid.



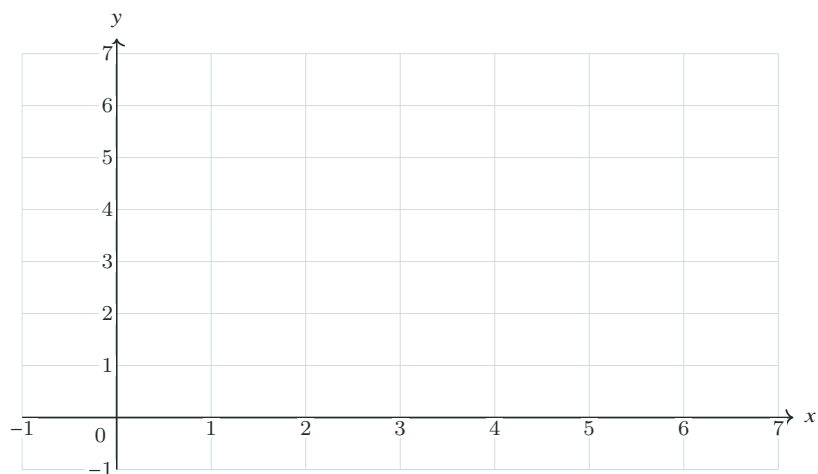
Q14 x and y are integers. Find how many points (x, y) satisfy $x \geq 0, y \geq 0, x + y \leq 3$. Mark these points on the grid.



Q15 x and y are integers. Find how many points (x, y) satisfy $x > 0, y > 0, x + y < 5$. Mark these points on the grid.

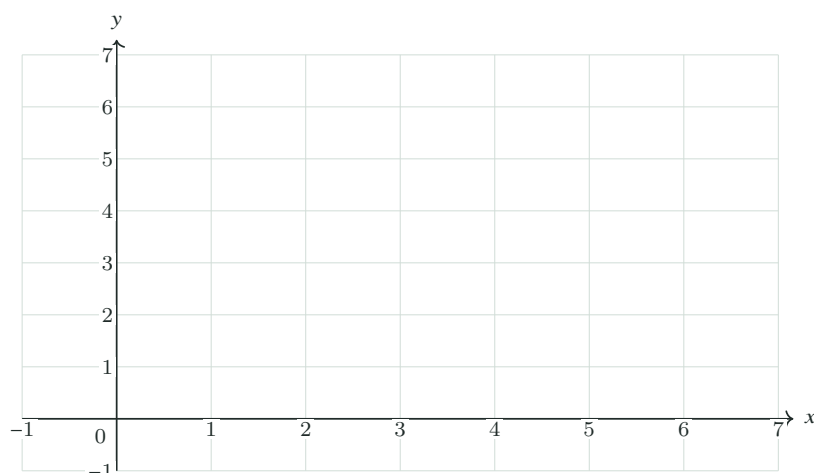


Q16 Find the greatest possible integer y if $x \geq 2$ and $2x + y \leq 11$.



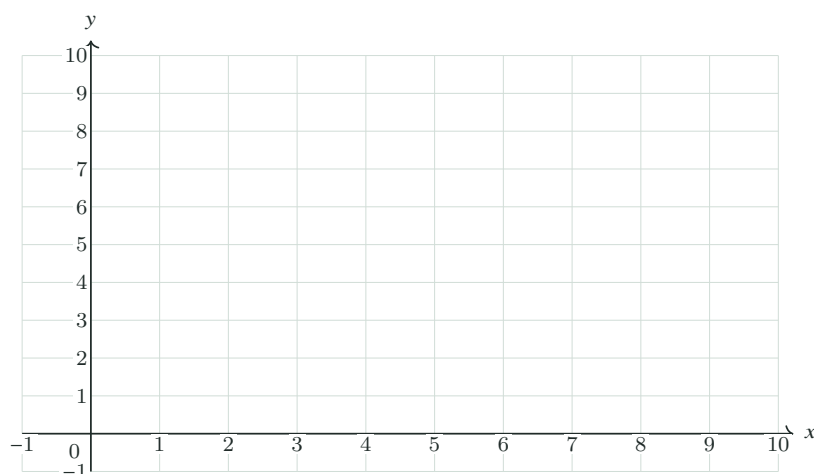
- Q17** Integers x and y satisfy $x \geq 0$, $y \geq 0$, $x + y \leq 6$ and $y \leq 2x$. Compare the greatest possible value of $x + 3y$ for each allowed y -value. What is the greatest value overall?

★★★★



- Q18** Tickets cost £3 for adults and £2 for children. A group has at least 5 adults, at least 4 children and a budget of £30. What is the largest possible group? Use x for the number of adults and y for the number of children.

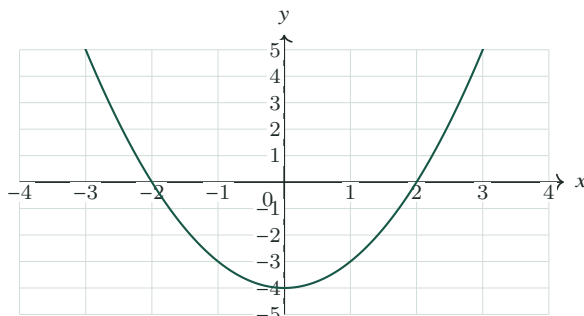
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4. Quadratic inequalities from graphs

Example Problem 5

Use the graph to solve $x^2 - 4 < 0$.



The idea

1. Find the roots, then identify the part of the curve below the axis.

Worked Solution

$$x = -2, 2$$

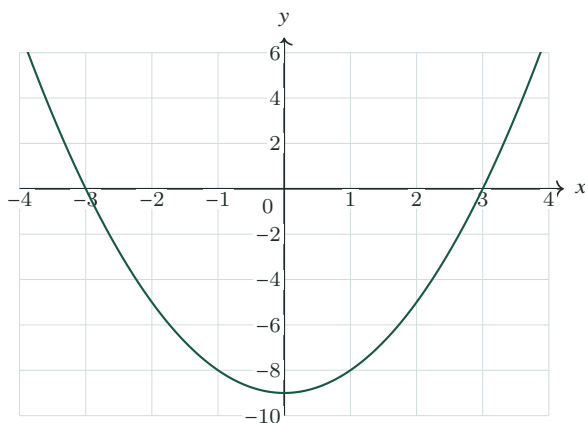
These are the boundary values.

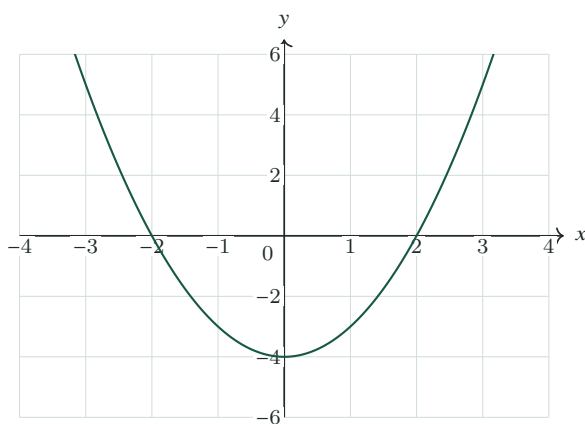
$$-2 < x < 2$$

The curve is below zero between the roots; exclude the endpoints.

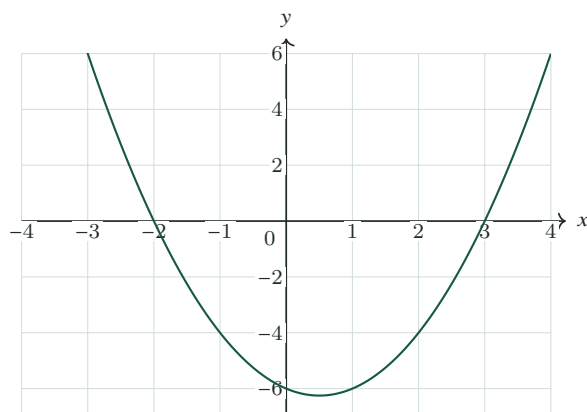
4 Quadratic inequalities from graphs

Q19 Use the graph to solve $x^2 - 9 < 0$.

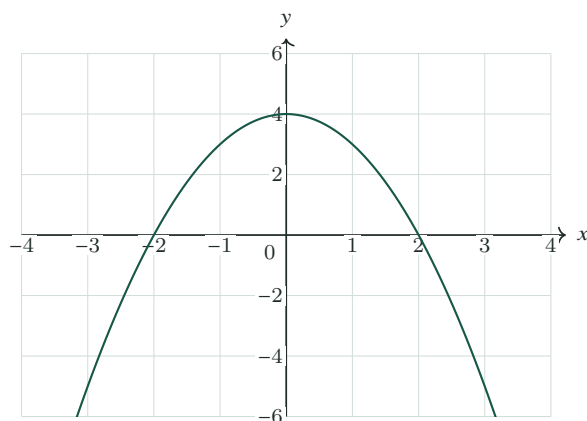
**Q20** Use the graph to solve $x^2 - 4 \geq 0$.



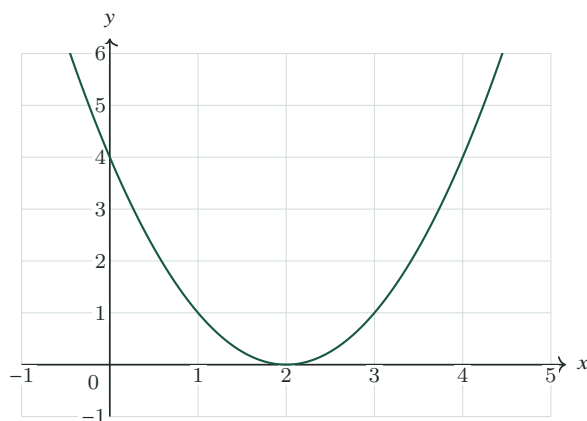
Q21 Use the graph to solve $x^2 - x - 6 \leq 0$.



Q22 Use the graph to solve $-x^2 + 4 > 0$.



Q23 Use the graph to solve $(x - 2)^2 \leq 0$.



Q24 Find all integer solutions of $x^2 - x - 6 < 0$.

Solving principles

1. Test a simple point

Use the origin when it is not on a boundary.

2. Check the edges

Keep solid and dashed boundaries consistent with the inequality signs.

3. List systematically

For integer points, fix x and list the permitted y -values.

4. Check the whole condition

Check that your answer satisfies every inequality.