

GCSE MATHS

Inequalities

Worked Solutions

Q1

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$$3x < 12$$

Subtract 2 from both sides.

$$x < 4$$

Divide both sides by 3.

Q2

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$$4x \geq 16$$

Add 7 to both sides.

$$x \geq 4$$

Divide both sides by 4.

Q3

**

(a) $-2x < 8$

Subtract 5 from both sides.

$$x > -4$$

Divide both sides by -2 . Reverse the inequality sign because the divisor is negative.

(b) -4 is excluded; values to its right are included.

Draw an open circle at -4 and an arrow to the right.

Q4

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$$-3x \geq 15$$

Subtract 1 from both sides.

$$x \leq -5$$

Divide both sides by -3 . Reverse the inequality sign because the divisor is negative.

Q5

*

$$5x \leq 15$$

Subtract 4 from both sides.

$$x \leq 3$$

Divide both sides by 5.

Q6

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$$-4x > 12$$

Add 3 to both sides.

$$x < -3$$

Divide both sides by -4 . Reverse the inequality sign because the divisor is negative.**Q7**

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$$2 - 3 < x \leq 9 - 3$$

Subtract three from all parts.

$$-1 < x \leq 6$$

Simplify.

Q8

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$$-6 < 2x \leq 6$$

Subtract one.

$$-3 < x \leq 3$$

Divide by two.

$$x = -2, -1, 0, 1, 2, 3$$

Include the upper endpoint but not the lower.

Q9

**

$$-6 \leq -2x < 6$$

Subtract ten.

$$3 \geq x > -3$$

Divide by -2 and reverse both signs.

$$-3 < x \leq 3$$

Write in increasing order.

Q10

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$$3x > 14 \Rightarrow x > \frac{14}{3}$$

Solve the inequality.

$$\text{smallest integer} = 5$$

Move to the next integer above $14/3$.

Q11

$$x \geq 3$$

Solve the first inequality.

$$x < 5$$

Solve the second inequality.

$$3 \leq x < 5$$

Take the overlap.

Q12

$$x < 7$$

Multiply the first inequality by three and add one.

$$x \geq -2$$

Solve the second.

$$-2 \leq x < 7$$

List the integers in this overlap.

Q13

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$$4 + 2d \leq 19$$

The cost must not exceed the budget.

$$d \leq 7.5$$

Subtract four and divide by two.

$$\text{greatest whole distance} = 7 \text{ km}$$

Use a whole number within the bound.

Q14

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$$2(x + 3) > 20$$

Form the perimeter inequality.

$$x + 3 > 10 \Rightarrow x > 7$$

Divide by two and subtract three.

Q15

$$\frac{62 + 70 + x}{3} \geq 75$$

Use the required mean.

$$x \geq 225 - 132 = 93$$

Multiply by three and subtract the known total.

$$93 \leq x \leq 100$$

A score cannot exceed 100.

Q16

$$30 + n < 12 + 3n$$

Require B to cost less.

$$n > 9$$

Rearrange.

$$n \geq 10 \text{ for whole visits}$$

Exclude the equal-cost case $n = 9$.

Q17

$$n < 9$$

Solve the two inequalities.

$$n > 5$$

$$n = 6, 7, 8$$

Take positive integers in the overlap.

Q18

$$2(x + 4 + x - 1) \leq 30$$

Form the perimeter condition.

$$4x + 6 \leq 30 \Rightarrow x \leq 6$$

Solve.

$$x - 1 > 0 \Rightarrow x > 1$$

Both dimensions must be positive.

Q19

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$$x = 2, 5$$

Find the roots.

$$(+)(-) < 0 \text{ when } 2 < x < 5$$

The factors have opposite signs only between the roots.

Q20

$$(x - 3)(x + 3) \geq 0$$

Factorise.

$$x \leq -3 \text{ or } x \geq 3$$

Both factors have the same sign outside the roots; include equality.

Q21

$$(x + 3)(x - 2) \leq 0$$

Factorise.

$$-3 \leq x \leq 2$$

The graph is on or below the x -axis between its roots. Include both roots because equality is allowed.

Q22

$$(2x + 1)(x - 3) > 0$$

Factorise.

$$x < -\frac{1}{2} \text{ or } x > 3$$

Take the outside intervals and exclude the roots.

Q23

$$(x - 5)(x + 1) < 0 \Rightarrow -1 < x < 5$$

Solve the quadratic inequality.

$$x \geq 2$$

Solve the linear condition.

$$2 \leq x < 5 \Rightarrow x = 2, 3, 4$$

Intersect the ranges, then select integers.

Q24

$$x(10 - x) \geq 21$$

Form the area inequality.

$$x^2 - 10x + 21 \leq 0$$

Move terms and reverse the sign after multiplying by -1 .

$$(x - 3)(x - 7) \leq 0$$

Factorise.

$$3 \leq x \leq 7$$

The range also gives positive side lengths.