

GCSE MATHS

Inequalities

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Preserving order

Add or subtract the same quantity on both sides. Multiplying or dividing by a negative number reverses the inequality sign.

Endpoints

$x \leq 3$ includes 3; $x < 3$ does not. On a number line, use a filled circle for an included endpoint and an open circle for an excluded endpoint.

Double inequalities

Apply each operation to all three parts, or solve two inequalities and take their overlap.

Quadratic signs

Find the roots and test the intervals between them. A positive coefficient of x^2 gives positive values outside two distinct roots and negative values between them.

1. Solving linear inequalities

Example Problem 1

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Solve $7 - 3x > 16$.

The idea

1. Isolate the variable term.
2. A negative divisor reverses the order.

Worked Solution

$$-3x > 9$$

Subtract seven.

$$x < -3$$

Divide by -3 and reverse the sign.

1 Solving linear inequalities

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Q1 Solve $3x + 2 < 14$.

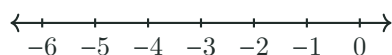
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Q2 Solve $4x - 7 \geq 9$.

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Q3 (a) Solve $-2x + 5 < 13$. (b) Show the solution on the number line.

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Q4 Solve $-3x + 1 \geq 16$.

**

Q5 Solve $5x + 4 \leq 19$.

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Q6 Solve $-4x - 3 > 9$.

**

2. Double inequalities and integer solutions

Example Problem 2

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List integer solutions of $1 \leq 2x + 3 < 11$.

The idea

1. Solve the double inequality first.
2. Then select integers respecting both endpoints.

Worked Solution

$$-2 \leq 2x < 8$$

Subtract three from all parts.

$$-1 \leq x < 4$$

Divide by two.

$$x = -1, 0, 1, 2, 3$$

Include -1 but exclude 4 .

2 Double inequalities and integer solutions

Q7 Solve $2 < x + 3 \leq 9$.

**

Q8 List the integer solutions of $-5 < 2x + 1 \leq 7$.

**

Q9 Solve $4 \leq 10 - 2x < 16$.

**

Q10 Find the smallest integer satisfying $3x - 4 > 10$.

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Q11 Find the values of x that satisfy both $2x + 1 \geq 7$ and $5 - x > 0$.

Q12 Find all integers satisfying $\frac{x-1}{3} < 2$ and $2x + 5 \geq 1$.

3. Forming inequalities

Example Problem 3

A club charges £8 plus £3 per session. Find the greatest whole number of sessions costing at most £30.

The idea

1. Use a variable for the number of sessions.
2. Keep the integer condition until after solving.

Worked Solution

$$8 + 3n \leq 30$$

$$n \leq \frac{22}{3}$$

$$n_{\max} = 7$$

The cost must be at most £30.

Solve.

Choose the greatest whole number allowed.

3 Forming inequalities

Q13 A taxi charges £4 plus £2 per kilometre. You have £19. Find the greatest whole number of kilometres you can afford.

**

Q14 A rectangle has width 3 cm and length x cm. Its perimeter is greater than 20 cm. Find the possible values of x .

**

Q15 Three test scores are 62, 70 and x , each out of 100. The mean must be at least 75. Find the possible values of x .

Q16 Plan A costs £12 plus £3 per visit. Plan B costs £30 plus £1 per visit. For how many visits is B cheaper?

Q17 A positive integer n satisfies $5n - 7 < 38$ and $3n + 2 > 17$. Find all possible n .

Q18 A rectangle has length $(x + 4)$ cm and width $(x - 1)$ cm. Its perimeter is at most 30 cm. Find the possible values of x .

4. Quadratic inequalities

Example Problem 4

Solve $x^2 - 3x - 10 \leq 0$.

The idea

1. Split the number line at the roots.
2. Test each interval and include roots for an inclusive upper bound.

Worked Solution

$$(x - 5)(x + 2) \leq 0$$

Factorise to locate the roots.

$$x = -2, 5$$

The sign can change only at a root.

$$-2 \leq x \leq 5$$

A test value $x = 0$ gives a negative product between the roots.

4 Quadratic inequalities

Q19 Solve $(x - 2)(x - 5) < 0$.

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Q20 Solve $x^2 - 9 \geq 0$.

Q21 Solve $x^2 + x - 6 \leq 0$.

Q22 Solve $2x^2 - 5x - 3 > 0$.

Q23 Find the integer solutions of $x^2 - 4x - 5 < 0$ that also satisfy $2x - 1 \geq 3$.

Q24 A rectangle has sides x cm and $(10 - x)$ cm. Its area is at least 21 cm^2 . Find all possible values of x .

Solving principles

1. Track sign changes

Reverse the inequality sign when dividing by a negative number.

2. Check whether integers are required

Solve the inequality first, then list whole-number values only if the question asks for integers.

3. Read the inequality words

At least means $\geq$, at most means $\leq$, and fewer than means $<$.

4. Test the intervals

Check a value from each region; do not assume a quadratic inequality has one continuous interval.