

GCSE MATHS

Indices and surds

Worked Solutions

Q1

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$$x^{4+7}$$

Add indices for the same base.

$$x^{11}$$

Add four and seven.

Q2

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$$y^{9-3}$$

Subtract the divisor index.

$$y^6$$

Subtract three from nine.

Q3

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$$3^2(a^2)^2(b^3)^2$$

Square every factor.

$$9a^4b^6$$

Multiply the indices inside each power.

Q4

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$$4x^{7-3}y^{2-5}$$

Divide coefficients and subtract indices.

$$\frac{4x^4}{y^3}$$

Move the negative power to the denominator.

Q5

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$$2^{x+1} = 2^5$$

Write 32 as a power of two.

$$x + 1 = 5$$

Equal powers of the same positive base have equal indices.

$$x = 4$$

Subtract one.

Q6

$$3^{2x-1} = 3^{3x-6}$$

Replace 27 by 3^3 .

$$2x - 1 = 3x - 6$$

Equate the indices.

$$x = 5$$

Collect x terms and constants.

Q7

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$$49^{1/2} = \sqrt{49}$$

A half power means a square root.

$$7$$

Use the positive square root.

Q8

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$$27^{2/3} = (\sqrt[3]{27})^2$$

Take the cube root before squaring.

$$3^2 = 9$$

The cube root is three.

Q9

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$$16^{-3/4} = \frac{1}{(\sqrt[4]{16})^3}$$

Use the negative sign for a reciprocal.

$$\frac{1}{2^3} = \frac{1}{8}$$

The fourth root of 16 is two.

Q10

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$$\left(\frac{9}{25}\right)^{1/2}$$

$$\frac{3}{5}$$

Invert the fraction to remove the negative index.

Take the square root of top and bottom.

Q11

$$(\sqrt{x})^3 = 125$$

Interpret the power as root then cube.

$$\sqrt{x} = 5$$

Take the cube root.

$$x = 25$$

Square both sides.

Q12

$$\frac{2^3 2^{2m}}{2^{n/2}}$$

$$2^{3+2m-n/2}$$

Replace each factor by a power of two.

Add numerator indices and subtract the denominator index.

Q13

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$$\sqrt{16 \times 3}$$

$$4\sqrt{3}$$

Find the largest square factor.

Take the square root of 16.

Q14

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$$5\sqrt{3} + 2\sqrt{3}$$

$$7\sqrt{3}$$

Extract the square factors 25 and four.

Add coefficients of the matching surds.

Q15

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$$15\sqrt{2} - 4\sqrt{2}$$

$$11\sqrt{2}$$

Simplify both roots before multiplying coefficients.

Subtract coefficients.

Q16

**

$$\sqrt{90}$$

$$\sqrt{9 \times 10} = 3\sqrt{10}$$

Multiply inside the root.

Extract the square factor nine.

Q17

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$$\sqrt{\frac{180}{5}}$$

$$\sqrt{36} = 6$$

Divide inside the square root.

Simplify the result of the division.

Q18

$$\sqrt{18} = 3\sqrt{2}$$

$$\sqrt{50} = 5\sqrt{2}$$

$$P = 2(3\sqrt{2} + 5\sqrt{2})$$

$$P = 16\sqrt{2} \text{ cm}$$

Simplify both side lengths.

Double the sum of adjacent sides.

Collect surds.

Q19

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$$(\sqrt{5})^2 - 2^2$$

$$5 - 4 = 1$$

Use the difference of squares.

Square each term.

Q20

$$9 + 3\sqrt{2} + 3\sqrt{2} + 2$$

$$11 + 6\sqrt{2}$$

Multiply every pair of terms.

Collect the rational terms and the surds.

Q21

$$\frac{7\sqrt{7}}{7}$$

$$\sqrt{7}$$

Multiply top and bottom by root seven.

Cancel seven.

Q22

$$\frac{3(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$\frac{6 - 3\sqrt{3}}{4 - 3}$$

$$6 - 3\sqrt{3}$$

Multiply top and bottom by $2 - \sqrt{3}$.

Use the difference of squares in the denominator.

The denominator equals one.

Q23

$$l = \frac{14}{\sqrt{11} - 1}$$

$$l = \frac{14(\sqrt{11} + 1)}{11 - 1} = \frac{7}{5}(\sqrt{11} + 1)$$

$$P = 2 \left(\frac{7}{5}(\sqrt{11} + 1) + \sqrt{11} - 1 \right)$$

$$P = \left(\frac{4}{5} + \frac{24}{5}\sqrt{11} \right) \text{ cm}$$

Divide the area by the width.

Multiply top and bottom by $\sqrt{11} + 1$.

Add length and width, then double.

Collect rational and surd terms.

Q24

$$a + b + 2\sqrt{ab} = 11 + 4\sqrt{6}$$

$$a + b = 11$$

$$ab = 24$$

$$(a, b) = (1, 24), (2, 12), (3, 8)$$

$$3 + 8 = 11$$

$$a = 8$$

$$b = 3$$

Expand the square.

Match rational parts and square the surd equality.

List positive factor pairs with product 24.

Only the pair 3 and 8 has sum 11.

Use a greater than b.