

GCSE MATHS

Indices and surds

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Index laws and equations	Q1–6	***	3
2. Fractional and negative indices	Q7–12	***	5
3. Simplifying surds	Q13–18	***	7
4. Surd expressions and reasoning	Q19–24	*****	9
Solving principles			12

Topic recap

Index laws

$a^m a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$ and $(a^m)^n = a^{mn}$. For division, $a \neq 0$.

Roots and reciprocals

For positive a , $a^{1/n} = \sqrt[n]{a}$ and $a^{-m} = 1/a^m$.

Square factors

$\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$. Extract a square factor before collecting surds.

Rationalising a denominator

$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$. Change the sign between the denominator terms and multiply top and bottom by that expression; the surd terms cancel.

1. Index laws and equations

Example Problem 1

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Simplify $\frac{(2x^3)^2}{4x}$, where $x \neq 0$.

The idea

1. Square the whole numerator before dividing.
2. Apply the index law only to powers with the same base.

Worked Solution

$$\frac{4x^6}{4x}$$

$$x^{6-1} = x^5$$

Square the coefficient and multiply the index by two.

Cancel four and subtract the indices.

1 Index laws and equations

Q1 Simplify $x^4 \times x^7$.

*

Q2 Simplify $y^9 \div y^3$, where $y \neq 0$.

*

Q3 Simplify $(3a^2b^3)^2$.

**

Q4 Simplify $\frac{12x^7y^2}{3x^3y^5}$, where $x, y \neq 0$. Use positive indices.

**

Q5 Solve $2^{x+1} = 32$.

**

Q6 Solve $3^{2x-1} = 27^{x-2}$.

2. Fractional and negative indices

Example Problem 2

**

Evaluate $81^{-3/4}$.

The idea

1. Separate the reciprocal from the fractional power.
2. Take the fourth root before cubing.

Worked Solution

$$\begin{aligned} & \frac{1}{81^{3/4}} \\ & \frac{1}{(\sqrt[4]{81})^3} \\ & \frac{1}{3^3} = \frac{1}{27} \end{aligned}$$

A negative power gives a reciprocal.

Take the fourth root first.

Cube the root.

2 Fractional and negative indices

Q7 Evaluate $49^{1/2}$.

**

Q8 Evaluate $27^{2/3}$.

**

Q9 Evaluate $16^{-3/4}$.

**

Q10 Evaluate $\left(\frac{25}{9}\right)^{-1/2}$.

**

Q11 Solve $x^{3/2} = 125$, where $x > 0$.

Q12 Given $a = 2^m$ and $b = 2^n$, write $\frac{8a^2}{\sqrt{b}}$ as a single power of 2.

3. Simplifying surds

Example Problem 3

**

Simplify $2\sqrt{27} + \sqrt{48}$.

The idea

1. Simplify each root to a multiple of root three.
2. Add the coefficients.

Worked Solution

$$2(3\sqrt{3}) + 4\sqrt{3}$$

$$6\sqrt{3} + 4\sqrt{3}$$

$$10\sqrt{3}$$

Extract square factors nine and 16.

Multiply the coefficient outside the first root.

Collect matching surds.

3 Simplifying surds

Q13 Simplify $\sqrt{48}$.

*

Q14 Simplify $\sqrt{75} + \sqrt{12}$.

**

Q15 Simplify $3\sqrt{50} - 2\sqrt{8}$.

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Q16 Simplify $\sqrt{6} \times \sqrt{15}$.

**

Q17 Simplify $\frac{\sqrt{180}}{\sqrt{5}}$.

**

Q18 A rectangle has sides $\sqrt{18}$ cm and $\sqrt{50}$ cm. Find its exact perimeter.

4. Surd expressions and reasoning

Example Problem 4

Rationalise the denominator of $\frac{4}{3-\sqrt{5}}$.

The idea

1. Use $3 + \sqrt{5}$ so the denominator becomes a difference of squares.
2. Expand the denominator and simplify the fraction.

Worked Solution

$$\frac{4(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})}$$
$$\frac{4(3 + \sqrt{5})}{9 - 5}$$
$$3 + \sqrt{5}$$

Multiply numerator and denominator by $3 + \sqrt{5}$.

The cross terms cancel.

Cancel four.

4 Surd expressions and reasoning

★★★★

Q19 Expand and simplify $(\sqrt{5} + 2)(\sqrt{5} - 2)$.

★★

Q20 Expand and simplify $(3 + \sqrt{2})^2$.

★★★

Q21 Rationalise the denominator of $\frac{7}{\sqrt{7}}$.

★★★

Q22 Rationalise the denominator of $\frac{3}{2+\sqrt{3}}$.

★★★

Q23 A rectangle has area 14 cm^2 and width $(\sqrt{11} - 1) \text{ cm}$. Find its exact perimeter in the form $a + b\sqrt{11}$.

Q24 Positive integers a and b satisfy $(\sqrt{a} + \sqrt{b})^2 = 11 + 4\sqrt{6}$, with $a > b$. Find a and b .

Solving principles

1. Identify the operation

Multiplication adds indices; a power of a power multiplies them.

2. Take the root first

For a fractional index, take the denominator root before applying the numerator power.

3. Keep exact form

Use surds rather than decimal approximations when an exact answer is requested.

4. Check signs and squares

Expand the squared brackets. The middle term in a squared bracket does not disappear.