

GCSE MATHS

Histograms

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Area represents frequency

In a histogram, frequency = class width $\times$ frequency density. The area of a bar represents the frequency; its height alone does not.

Unequal widths

A class from 10 to 25 has width 15. With frequency 30, its density is $30/15 = 2$.

Estimating inside a class

Use the relevant fraction of its width. This assumes values are evenly spread within that class.

Grouped mean

Use class midpoints as representative values. Multiply each midpoint by its frequency, add the products, then divide by total frequency.

A calculator may be used.

1. Class widths and frequency density

Example Problem 1

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A class covers $12 < t \leq 20$ and has frequency 28. Find its frequency density.

The idea

1. Histogram height must account for unequal class widths.
2. Choose the height so width $\times$ height reproduces the frequency.

Worked Solution

$$w = 20 - 12 = 8$$

Find the width.

$$d = 28/8 = 3.5$$

Divide frequency by width.

1 Class widths and frequency density

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Q1 The class $0 < t \leq 5$ has frequency 20. Find its frequency density.

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Q2 The class $10 < t \leq 25$ has frequency 45. Find its frequency density.

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Q3 The class $20 < t \leq 30$ has frequency 18. Find its frequency density.

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Q4 A histogram bar has width 12 and frequency density 2.5. Find its frequency.

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Q5 A class has frequency 42 and frequency density 3.5. Its lower boundary is 18. Find its upper boundary.

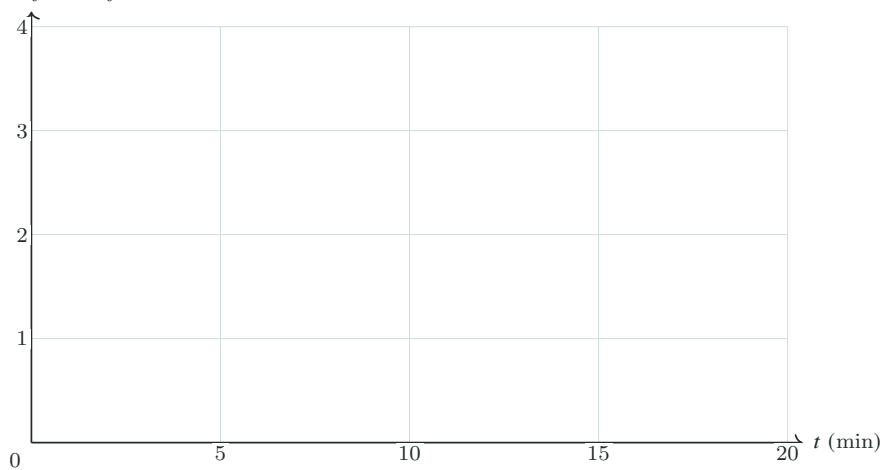
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Q6 The table shows journey times. Draw a histogram on the axes.

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Time t (minutes)	Frequency
$0 < t \leq 5$	10
$5 < t \leq 15$	20
$15 < t \leq 20$	15

Frequency density

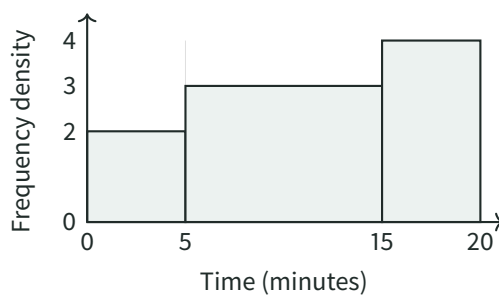


2. Reading histogram frequencies

Example Problem 2

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Find the total frequency shown.



The idea

1. Read each class width from the horizontal axis.
2. Multiply each width by the corresponding height.

Worked Solution

$$f_1 = 5(2) = 10$$

$$f_2 = 10(3) = 30$$

$$f_3 = 5(4) = 20$$

$$N = 10 + 30 + 20 = 60$$

Find the first two areas.

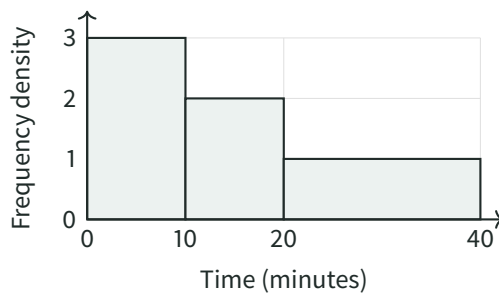
Find the last area.

Add them.

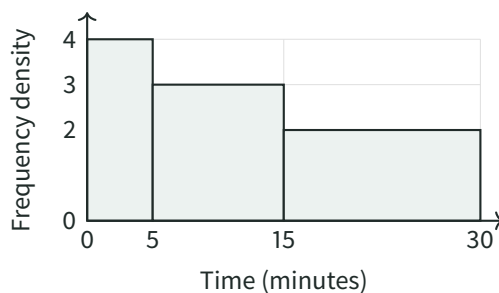
2 Reading histogram frequencies

Q7 The histogram shows journey times. How many journeys took more than 20 minutes?

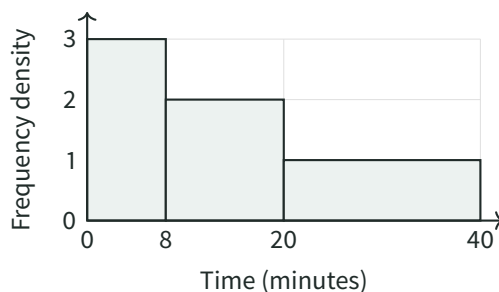
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**Q8** Find the total frequency represented by the histogram.

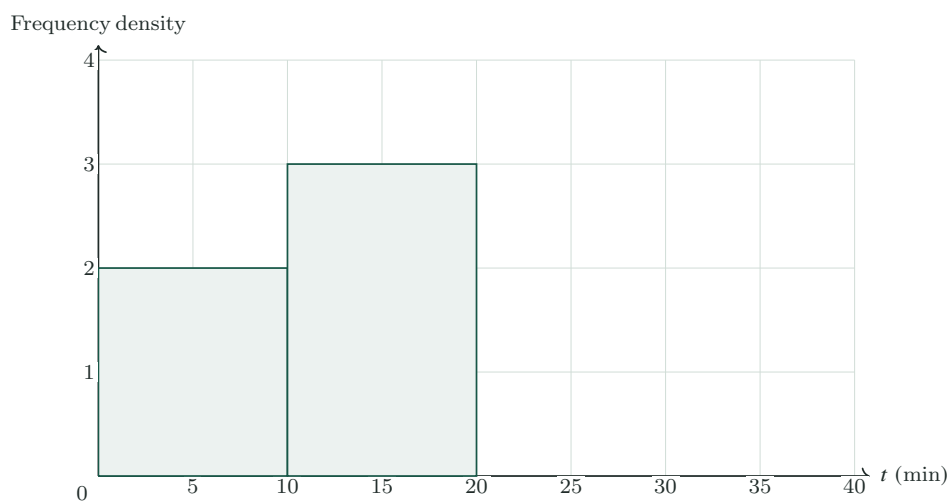
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**Q9** Which classes contain the greatest number of observations? State this frequency.

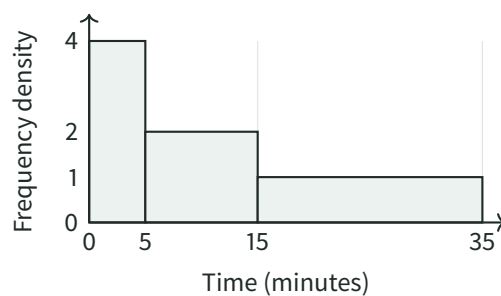
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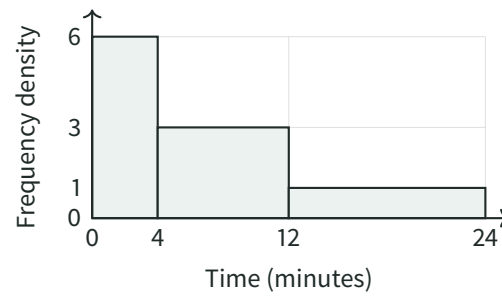
Q10 The histogram represents 80 journey times. The bar for $20 < t \leq 40$ is missing. Calculate its frequency density and draw the missing bar. ***



Q11 In the histogram, what fraction of observations lie in $5 < t \leq 15$? ***



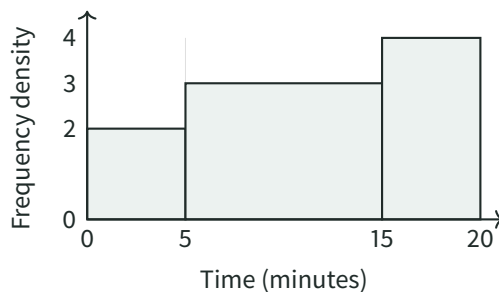
Q12 Find the ratio of the frequency in the first class to the frequency in the last class.



3. Estimating from grouped data

Example Problem 3

Estimate the median from the histogram. Assume an even spread within each class.



The idea

1. Find the class containing the middle observation.
2. Move through that class in the same fraction as its frequency.

Worked Solution

$$N = 60$$

$$N/2 = 30$$

$$\text{median} \approx 5 + \frac{30 - 10}{30}(10)$$

$$\text{median} \approx 11.7 \text{ min (1 d.p.)}$$

Locate the middle position.

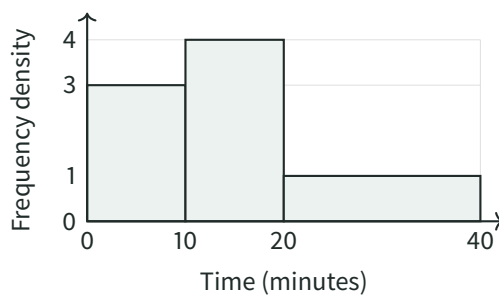
The first class contains 10; move 20 of the next 30 into its width.

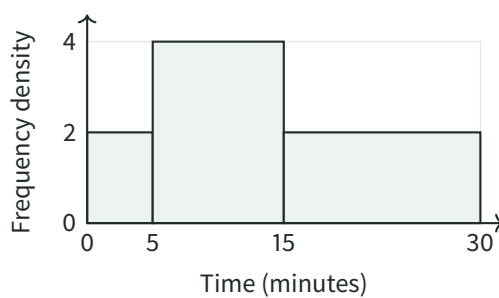
Evaluate.

3 Estimating from grouped data

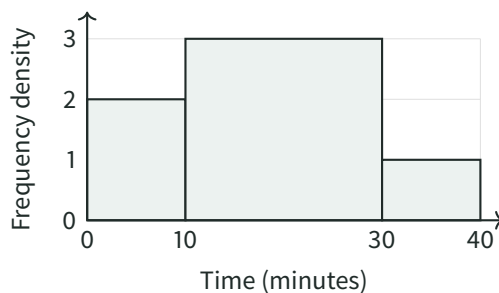
Q13 Estimate how many observations have $t \leq 15$. Assume an even spread within each class.

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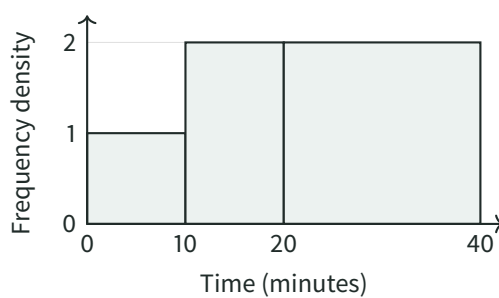
**Q14** Estimate the median time from the histogram. Assume an even spread within each class.



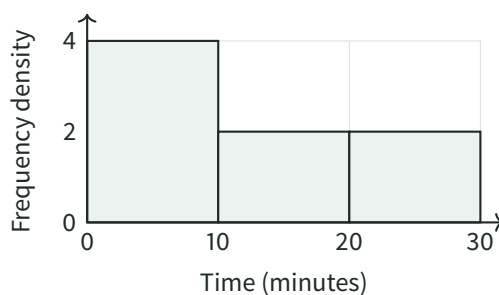
Q15 Estimate the mean time from the histogram, to 1 decimal place.



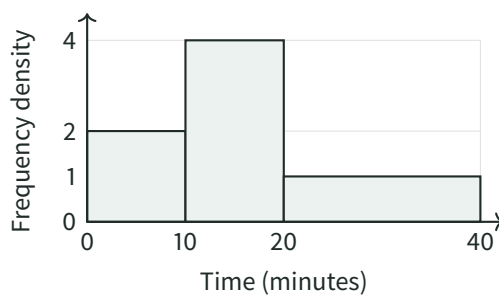
Q16 Estimate the percentage of observations with $t > 25$, to 1 decimal place. Assume an even spread within each class.



- Q17** Estimate the lower quartile time from the histogram. Assume an even spread within each class. ***



- Q18** Estimate the interquartile range from the histogram. Assume an even spread within each class. ***



4. Missing information and comparisons

Example Problem 4

Two classes have widths 5 and 15 and densities k and $2k$. Their total frequency is 70. Find k .

The idea

1. Convert each unknown height into an area.
2. Use the total frequency to form an equation.

Worked Solution

$$5k + 15(2k) = 70$$

Add bar areas.

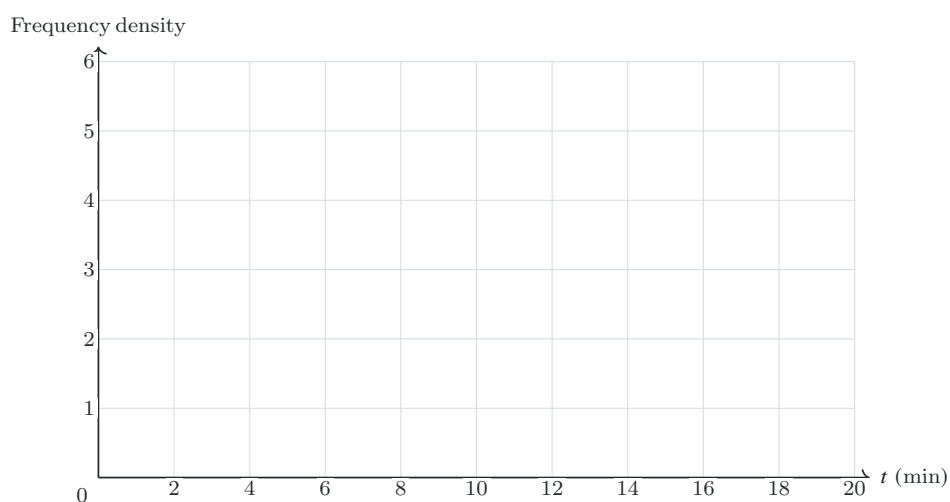
$$35k = 70 \Rightarrow k = 2$$

Collect terms and divide.

4 Missing information and comparisons

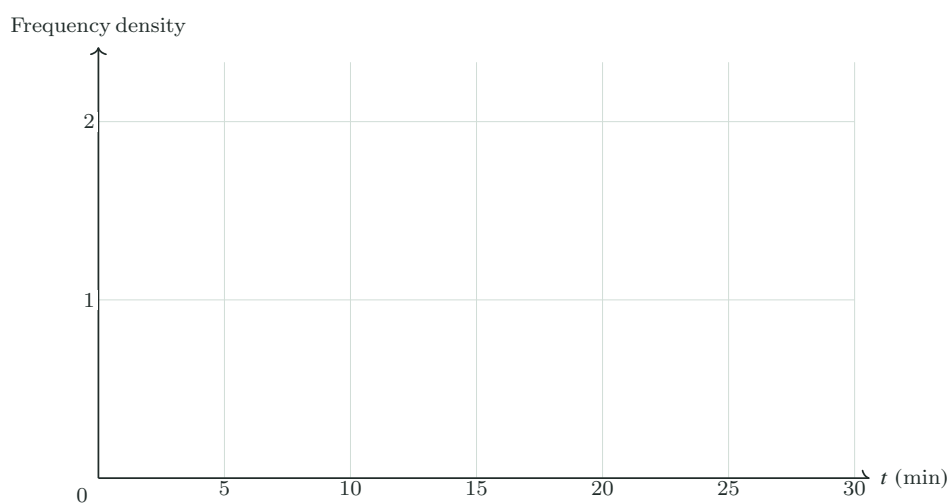
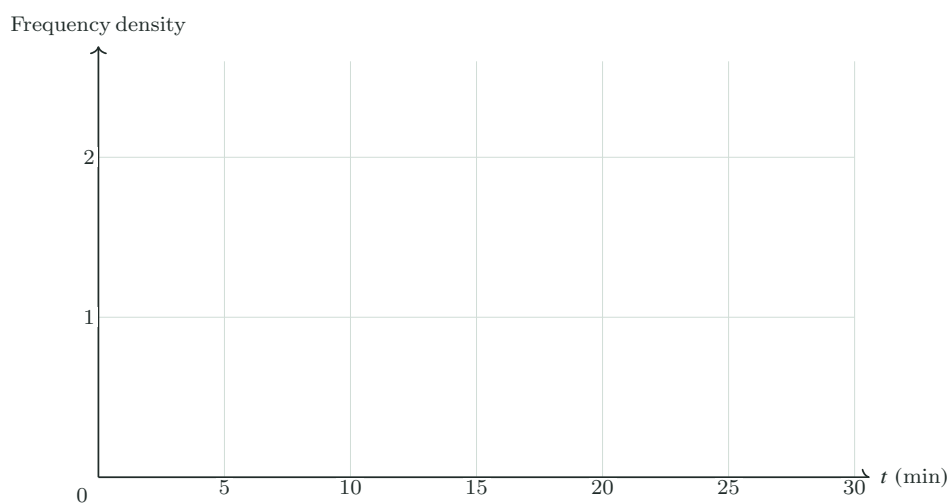
Q19 The table shows journey times. Draw a histogram. Then find how many journeys took more than 8 minutes.

Time t (minutes)	Frequency
$0 < t \leq 8$	20
$8 < t \leq 20$	60



Q20 A histogram is drawn with 1 cm representing 5 minutes horizontally. Vertically, 1 cm represents density 2. A bar has paper area 6 cm^2 . Find its frequency.

- Q21** The classes $0 < t \leq 10$ and $10 < t \leq 30$ have frequencies 16 and 24. They are merged into one class. Find the new bar height as a frequency density. Draw the original two bars and the merged bar on the separate axes.



Q22 The classes $0 < t \leq 10$ and $10 < t \leq 30$ have densities a and b . There are 100 observations in total and 40% lie in the first class. Find $a : b$.

Q23 A histogram has classes $0 < t \leq 10$, $10 < t \leq 20$, $20 < t \leq 40$ with densities 2, k , 1. An estimated 60% of observations have $t \leq 20$. Find k .

Q24 A histogram has classes $0 < t \leq 10$, $10 < t \leq 20$, $20 < t \leq 40$ with densities 2, k , 1. Its estimated mean, using class midpoints, is 17.5 minutes. Find k .

Solving principles

1. Read both axes

Bar area represents frequency. If the density scale is missing, use a bar with known frequency to find it.

2. Use area

Multiply width by height to compare frequencies.

3. Keep estimates explicit

A grouped table does not reveal every original value. Use within-class estimates only when asked.

4. Check the total frequency

Add bar frequencies and compare with the stated sample size.