

GCSE MATHS

Functions

Worked Solutions

Q1

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$$f(5) = 4(5) - 7 = 13$$

Replace x by 5.**Q2**

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$$g(-4) = (-4)^2 + 3(-4)$$

Use brackets around the negative input.

$$g(-4) = 16 - 12 = 4$$

Square before adding.

Q3

★★

$$5 - 2x = 17$$

Set the function equal to its output.

$$-2x = 12$$

Subtract five and divide by -2 .

$$x = -6$$

Q4

★★

$$x^2 - 4 = 21$$

Set the output.

$$x^2 = 25$$

Include both square roots.

$$x = \pm 5$$

Q5

★★

$$f(2x - 1) = 3(2x - 1) + 2$$

Replace the input as a whole.

$$f(2x - 1) = 6x - 1$$

Expand and collect.

Q6

★★

$$2(x + 1)^2 - 1 - (2x^2 - 1)$$

Subtract the whole second expression.

$$2x^2 + 4x + 2 - 1 - 2x^2 + 1 = 4x + 2$$

Expand and cancel.

Q7

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(a) $g(4) = 16$

Apply g first.

$$f(16) = 2(16) + 3 = 35$$

Apply f to the result.

(b) $f(4) = 11$

Apply f first.

$$g(11) = 11^2 = 121$$

Square its output.

Q8

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$$fg(x) = 3(5x + 1) - 2$$

Put $g(x)$ into f .

$$fg(x) = 15x + 1$$

Expand and collect.

Q9

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$$gf(x) = (x^2 + 1) - 3$$

Apply g to the output of f .

$$gf(x) = x^2 - 2$$

Collect constants.

Q10

$$2x^2 - 1 = 17$$

Form the composite function.

$$x^2 = 9$$

Rearrange and take both roots.

$$x = \pm 3$$

Q11

$$fg(x) = 3x + 2$$

Write both orders.

$$gf(x) = 3x + 6$$

$$3x + 2 = 3x + 6 \Rightarrow 2 = 6$$

The equation is impossible.

Q12

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$$y = 5x - 4$$

$$x = \frac{y + 4}{5}$$

$$f^{-1}(x) = \frac{x + 4}{5}$$

Write the output as y .

Reverse subtracting four, then multiplying by five.

Rename the inverse input x .

Q13

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$$3y = x + 7$$

$$x = 3y - 7$$

$$g^{-1}(x) = 3x - 7$$

Multiply by three.

Subtract seven.

Rename the input.

Q14

$$7 - 2x = 9$$

$$-2x = 2$$

$$x = -1$$

Find the input to f giving output 9.

Rearrange.

Q15

(a) $y = x^2 + 4 \Rightarrow x = \sqrt{y - 4}$

$$f^{-1}(x) = \sqrt{x - 4}$$

(b) $x - 4 \geq 0 \Rightarrow x \geq 4$

Use the non-negative root because x is non-negative.

Write the inverse using x as its input.

The expression inside the square root must be zero or positive.

Q16

$$y(x - 2) = 6$$

$$x = 2 + \frac{6}{y}$$

$$g^{-1}(x) = 2 + \frac{6}{x}$$

$$x \neq 0$$

Multiply by the denominator.

Divide by y and add two.

Zero is never an output of g .

Q17

$$fg(x) = 3(2x - 5) + 1 = 6x - 14$$

$$y = 6x - 14 \Rightarrow x = \frac{y + 14}{6}$$

$$(fg)^{-1}(x) = \frac{x + 14}{6}$$

Form the composition first.

Reverse the composite operations.

Rename the inverse input.

Q18

$$(2x + 3)^2 = 2x^2 + 3$$

$$2x^2 + 12x + 6 = 0$$

$$x^2 + 6x + 3 = 0$$

$$x = -3 \pm \sqrt{6}$$

Write both compositions.

Expand and collect.

Divide by two.

Use the quadratic formula.

Q19

$$2a + b = 9$$

$$5a + b = 21$$

$$3a = 12 \Rightarrow a = 4$$

$$b = 9 - 8 = 1$$

Use the two input-output pairs.

Subtract the equations.

Substitute.

Q20

$$ff(3) = 2(6 + k) + k$$

$$12 + 3k = 27 \Rightarrow k = 5$$

Apply f twice.

Collect the three copies of k .

Q21

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$$ff(x) = a^2x + ab + b$$

$$a^2 = 1$$

$$b(a + 1) = 0$$

$$a = 1 \Rightarrow b = 0$$

$$a = -1 \Rightarrow b \text{ is unrestricted}$$

Expand the composition.

Match the coefficient of x and the constant.

Use $a + 1 = 2$.

The constant condition is then automatic.

Q22

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$$fg(x) = \frac{1}{2x - 1}$$

$$2x - 1 \neq 0 \Rightarrow x \neq \frac{1}{2}$$

Insert $2x$ as the input.

The composite denominator must be non-zero.

Q23

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(a) $y = (x - 3)^2 + 4$

$$x - 3 = \sqrt{y - 4}$$

$$f^{-1}(x) = 3 + \sqrt{x - 4}$$

(b) $x - 4 \geq 0 \Rightarrow x \geq 4$

Complete the square.

The restriction x is at least 3 selects the non-negative root.

Write the inverse using x as its input.

The expression inside the square root must be zero or positive.