

GCSE MATHS

Fractions, decimals and recurring decimals

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Addition and subtraction

Use a common denominator: $\frac{2}{3} + \frac{1}{4} = \frac{8}{12} + \frac{3}{12} = \frac{11}{12}$.

Multiplication and division

Multiply numerators and denominators; cancel common factors first. To divide by a non-zero fraction, multiply by its reciprocal.

Mixed numbers

Convert a mixed number before multiplying: $2\frac{1}{3} = \frac{7}{3}$.

Recurring notation

A bar marks the repeating block: $0.\overline{27} = 0.272727 \dots$. Multiply by powers of ten so identical recurring tails can be subtracted.

1. Fraction arithmetic

Example Problem 1

**

Calculate $1\frac{1}{2} + \frac{5}{6}$.

The idea

1. Convert the mixed number, then match denominators.

Worked Solution

$$1\frac{1}{2} = \frac{3}{2} = \frac{9}{6}$$

$$\frac{9}{6} + \frac{5}{6} = \frac{14}{6} = \frac{7}{3}$$

Use an improper fraction and a common denominator.

Add and simplify.

1. Fraction arithmetic

Example Problem 2

**

Calculate $\frac{7}{12} \div \frac{14}{15}$.

The idea

1. Dividing by a fraction reverses multiplication by that fraction, so multiply by its reciprocal.

Worked Solution

$$\begin{aligned}\frac{7}{12} \times \frac{15}{14} \\ \frac{1}{4} \times \frac{5}{2} = \frac{5}{8}\end{aligned}$$

Turn the divisor over and multiply.

Cancel 7 against 14 and 3 against 15 and 12.

1 Fraction arithmetic

**

Q1 Calculate $\frac{2}{3} + \frac{5}{12}$. Give a fraction in its simplest form.

*

Q2 Calculate $\frac{7}{8} - \frac{1}{3}$. Give a fraction in its simplest form.

*

Q3 Calculate $\frac{9}{14} \times \frac{7}{15}$. Give a fraction in its simplest form.

**

Q4 Calculate $\frac{5}{6} \div \frac{10}{9}$. Give a fraction in its simplest form.

**

Q5 Calculate $2\frac{1}{3} \times 1\frac{1}{2}$. Give a fraction in its simplest form.

**

Q6 Calculate $\frac{3}{4} - \frac{2}{3} \div 4$. Give a fraction in its simplest form.

**

2. Fractions, decimals and percentages

Example Problem 3

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Convert $\frac{3}{16}$ to (a) a decimal; (b) a percentage.

The idea

1. A fraction is a division; percent means out of 100.

Worked Solution

(a) $3 \div 16 = 0.1875$

Divide numerator by denominator.

(b) $0.1875 \times 100 = 18.75\%$

Convert the decimal to a percentage.

2. Fractions, decimals and percentages

Example Problem 4

Explain why $\frac{3}{20}$ terminates and $\frac{1}{6}$ recurs.

The idea

1. A terminating decimal can be written with denominator 10, 100, 1000 or another power of ten.

Worked Solution

$$20 = 2^2 \times 5$$

$$\frac{3}{20} = \frac{15}{100}$$

$$6 = 2 \times 3$$

$$10^n = 2^n 5^n$$

A power of ten supplies every prime factor needed.

The fraction is in simplest form and its denominator contains 3.

No power of ten supplies a factor of 3; the decimal cannot terminate.

2 Fractions, decimals and percentages

Q7 Write 0.375 as a fraction in simplest form.

*

Q8 Write $\frac{7}{20}$ as (a) a decimal; (b) a percentage.

*

Q9 Write 12.5% as a fraction in simplest form.

**

Q10 Put $\frac{5}{8}$, 0.62 and 63% in increasing order.

**

Q11 (a) Find $\frac{3}{5}$ of 240. (b) Express this result as a percentage of 180.

**

Q12 Without long division, explain why $\frac{7}{40}$ has a terminating decimal but $\frac{7}{30}$ does not.

3. Recurring decimals as fractions

Example Problem 5

Convert $0.2\overline{7}$ to a fraction.

The idea

1. Move the non-recurring digits before the decimal point.
2. Move one complete recurring block further, then subtract.

Worked Solution

$$x = 0.2777 \dots$$

Only the 7 repeats.

$$10x = 2.777 \dots$$

Multiply so the recurring tails match.

$$100x = 27.777 \dots$$

$$90x = 25$$

Subtract the first equation from the second.

$$x = \frac{25}{90} = \frac{5}{18}$$

Simplify.

3 Recurring decimals as fractions

Q13 Convert $0.\overline{3}$ to a fraction in simplest form.

**

Q14 Convert $0.\overline{27}$ to a fraction in simplest form.

**

Q15 Convert $0.\overline{18}$ to a fraction in simplest form.

**

Q16 Convert $0.1\overline{6}$ to a fraction in simplest form.

Q17 Convert $0.24\overline{5}$ to a fraction in simplest form.

Q18 Convert $1.\overline{23}$ to a fraction in simplest form.

**

4. Fraction problems and reasoning

Example Problem 6

A rope loses $\frac{1}{4}$ of its length, then $\frac{1}{3}$ of the remaining length. What fraction of the original remains?

The idea

1. Apply each fraction to its stated whole.

Worked Solution

$$1 - \frac{1}{4} = \frac{3}{4}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$

Find the first remainder.

Retain two thirds of the remainder.

4 Fraction problems and reasoning

Q19 A tank is $\frac{3}{5}$ full. After 18 litres are removed, it is $\frac{3}{8}$ full. Find its capacity.

Q20 A pupil spends $\frac{1}{3}$ of their money, then $\frac{2}{5}$ of the remainder. What fraction of the original amount remains?

Q21 One third of a class walk to school and one quarter cycle. The remaining 15 pupils use a bus. How many pupils are in the class?

Q22 Find $0.\overline{27} + 0.\overline{36}$ as a fraction in simplest form.

Q23 A fraction is unchanged when 2 is added to its numerator and 3 to its denominator. Its denominator is 15 greater than its numerator. Find the numerator and denominator.

Q24 A fraction in simplest form has denominator 84. After multiplying by $\frac{3}{7}$, the result has a terminating decimal. Find all possible positive numerators smaller than 84, or explain why there are none.

Solving principles

1. Track the whole

A fraction of what remains is not the same fraction of the original amount.

2. Keep answers exact

Use fractions until a decimal approximation is requested.

3. Align the recurring tails

Subtract expressions whose infinite decimal tails match.

4. Check the interval

A fraction or decimal answer should have the expected size.