

GCSE MATHS

Factors, multiples and primes

Worked Solutions

Q1

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$$84 = 2^2 \times 3 \times 7$$

Divide by primes until every factor is prime.

Q2

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$$180 = 2^2 \times 3^2 \times 5$$

Divide by primes until every factor is prime.

Q3

*

$$360 = 2^3 \times 3^2 \times 5$$

Divide by primes until every factor is prime.

Q4

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$$91 = 7 \times 13$$

Both factors are prime.

Q5

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$$18 = 2 \times 3^2$$

Compare the prime factors.

$$N/18 = 20$$

The quotient is an integer.

Q6

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$$4 = 2^2$$

The square of the smallest prime has three factors.

$$1, 2, 4$$

The integers 1, 2 and 3 have fewer than three factors.

Q7

**

$$48 = 2^4 \times 3$$

$$72 = 2^3 \times 3^2$$

$$\text{HCF} = 24$$

Write each number as a product of primes.

Take the smaller exponent for every shared prime.

Q8

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$$84 = 2^2 \times 3 \times 7$$

$$126 = 2 \times 3^2 \times 7$$

$$\text{HCF} = 42$$

Write each number as a product of primes.

Take the smaller exponent for every shared prime.

Q9

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$$150 = 2 \times 3 \times 5^2$$

$$210 = 2 \times 3 \times 5 \times 7$$

$$\text{HCF} = 30$$

Write each number as a product of primes.

Take the smaller exponent for every shared prime.

Q10

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$$72 = 2^3 \times 3^2$$

$$120 = 2^3 \times 3 \times 5$$

$$\text{HCF} = 2^3 \times 3 = 24$$

The number of bags divides both totals.

Use the smaller shared prime indices.

Q11

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$$\text{HCF} = 2^2 \times 3^2$$

$$\text{HCF} = 36$$

Keep only shared primes, with smaller indices.

Multiply.

Q12

$$60 = 12(5)$$

$$n = 12k$$

$$k > 5$$

$$k = 6 \text{ is coprime to } 5$$

$$n = 72$$

The remaining factors must be coprime.

Choose the first permitted integer.

Multiply by the HCF.

Q13

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$$12 = 2^2 \times 3$$

$$18 = 2 \times 3^2$$

$$\text{LCM} = 36$$

$$36/12 = 3$$

$$36/18 = 2$$

Write both prime factorisations.

Take the larger exponent of every prime in the two factorisations.

Both numbers divide the result.

Q14

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$$24 = 2^3 \times 3$$

$$40 = 2^3 \times 5$$

$$\text{LCM} = 120$$

$$120/24 = 5$$

$$120/40 = 3$$

Write both prime factorisations.

Take the larger exponent of every prime in the two factorisations.

Both numbers divide the result.

Q15

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$$45 = 3^2 \times 5$$

$$60 = 2^2 \times 3 \times 5$$

$$\text{LCM} = 180$$

$$180/45 = 4$$

$$180/60 = 3$$

Write both prime factorisations.

Take the larger exponent of every prime in the two factorisations.

Both numbers divide the result.

Q16

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$$18 = 2 \times 3^2$$

$$24 = 2^3 \times 3$$

$$\text{LCM} = 2^3 \times 3^2 = 72 \text{ s}$$

$$10:00:00 + 72 \text{ s} = 10:01:12$$

Find the first shared repeat.

Add 1 minute 12 seconds.

Add the interval to the starting time.

Q17

$$8 = 2^3$$

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$N = 2^3 \times 3 \times 5 = 120$$

List the prime powers needed.

Use each highest required exponent once.

Q18

$$30n = 6(180)$$

$$n = 36$$

Use the $\text{HCF} \times \text{LCM}$ product relation.

Divide by 30 and check both conditions.

Q19

$$180 = 2^2 \times 3^2 \times 5$$

$$k = 5$$

$$180k = 900 = 30^2$$

Square indices must be even.

Supply one more factor of 5.

Q20

$$72 = 2^3 \times 3^2$$

$$k = 3$$

$$72k = 216 = 6^3$$

Cube indices must be multiples of three.

Supply one factor of 3.

Q21

$$360 = 2^3 \times 3^2 \times 5$$

$$k = 2 \times 5 = 10$$

$$360/k = 36$$

Remove the unmatched factors.

Any square quotient needs at least these removals.

Q22

$$72 = 2^3 \times 3^2$$

$$360 = 2^3 \times 3^2 \times 5$$

5 divides neither N nor 72

Compare the prime factors of the proposed LCM.

An LCM cannot introduce a new prime.

Q23

$$a = 6u$$

$$b = 6v$$

$$uv = 72/6 = 12$$

$$(u, v) = (1, 12), (2, 6), (3, 4)$$

$$(a, b) = (6, 72), (18, 24)$$

The integers u and v must be coprime.

Use the product relation.

List factor pairs.

Reject $(2, 6)$, which is not coprime.

Q24

$$N = 2^a 3^b$$

$$a - 2 \text{ even}$$

$$a - 1 \text{ a multiple of 3}$$

$$b - 1 \text{ even}$$

$$b - 2 \text{ a multiple of 3}$$

$$N = 2^4 \times 3^5 = 3888$$

$$N/12 = 324 = 18^2$$

$$N/18 = 216 = 6^3$$

No other prime is needed for the smallest value.

The smallest exponent of 2 is 4.

The smallest exponent of 3 is 5.

Check the two quotients.

Both required perfect-power conditions hold.