

GCSE MATHS

Factors, multiples and primes

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Prime factors

A prime number has exactly two positive factors. Divide repeatedly by primes; write repeated factors using powers.

HCF and LCM

In prime-factor form, the HCF uses the smaller exponent of each shared prime. The LCM uses the larger exponent of every prime present. Two numbers are coprime when their HCF is 1.

Perfect powers

In a square, every prime exponent is even. In a cube, every prime exponent is a multiple of three.

Product connection

For two positive integers, their product equals their HCF multiplied by their LCM.

1. Prime factorisation

Example Problem 1

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Write 252 as a product of prime factors.

The idea

1. Divide by a prime that goes into the number.
2. Continue until every factor is prime.

Worked Solution

$$252 = 2(126) = 2^2(63)$$

Divide twice by 2.

$$252 = 2^2 \times 3^2 \times 7$$

Factor 63 into primes.

1 Prime factorisation

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Q1 Write 84 as a product of prime factors.

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Q2 Write 180 as a product of prime factors.

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Q3 Write 360 as a product of prime factors.

*

Q4 Write 91 as a product of prime factors.

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Q5 $N = 2^3 \times 3^2 \times 5$. Is N divisible by 18? Explain.

**

Q6 Find the smallest positive integer with exactly three positive factors.

**

2. Highest common factors

Example Problem 2

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Find the HCF of 60 and 84.

The idea

1. A common divisor can use only prime factors present in both numbers.

Worked Solution

$$60 = 2^2 \times 3 \times 5$$

$$84 = 2^2 \times 3 \times 7$$

$$\text{HCF} = 2^2 \times 3 = 12$$

Write prime factors.

Choose the smaller shared indices.

2 Highest common factors

Q7 Find the HCF of 48 and 72.

**

Q8 Find the HCF of 84 and 126.

**

Q9 Find the HCF of 150 and 210.

**

Q10 72 red beads and 120 blue beads are put into bags so that every bag has the same number of red beads and the same number of blue beads, using every bead. Find the greatest number of bags.

**

Q11 $A = 2^4 \times 3^2 \times 5$ and $B = 2^2 \times 3^3 \times 7$. Find their HCF.

**

Q12 Two positive integers have HCF 12. One is 60. Give the smallest possible value of the other if it is greater than 60.

3. Lowest common multiples

Example Problem 3

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Find the LCM of 18 and 30.

The idea

1. Use the largest required exponent for each prime.

Worked Solution

$$18 = 2 \times 3^2$$

$$30 = 2 \times 3 \times 5$$

$$\text{LCM} = 2 \times 3^2 \times 5 = 90$$

Write prime factors.

Include enough copies of each prime to contain either number.

3. Lowest common multiples

Example Problem 4

Two positive integers have HCF 4 and LCM 60. One is 12. Find the other.

The idea

1. For two positive integers, HCF times LCM equals their product.

Worked Solution

$$12n = 4(60)$$

Equate the two products.

$$n = 20$$

Divide by 12.

$$\text{HCF}(12, 20) = 4$$

Check the shared prime factor.

$$\text{LCM}(12, 20) = 60$$

Check the least common multiple too.

3 Lowest common multiples

Q13 Find the LCM of 12 and 18.

**

Q14 Find the LCM of 24 and 40.

**

Q15 Find the LCM of 45 and 60.

**

Q16 Two lights flash every 18 seconds and 24 seconds. They flash together at 10:00:00. When do they next flash together?

**

Q17 Find the smallest positive integer divisible by 8, 12 and 15.

Q18 Two positive integers have HCF 6 and LCM 180. One integer is 30. Find the other.

4. Powers and integer reasoning

Example Problem 5

Find the smallest positive k such that $200k$ is a square.

The idea

1. In a square, prime factors pair up.

Worked Solution

$$200 = 2^3 \times 5^2$$

Identify odd indices.

$$k = 2$$

Add one factor of 2.

$$200k = 400 = 20^2$$

4. Powers and integer reasoning

Example Problem 6

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Find the smallest positive integer N such that $N/8$ is a square number and $N/12$ is a cube number.

The idea

1. Write N using powers of 2 and 3, the primes in 8 and 12.
2. Find the smallest exponent of each prime that meets both conditions.

Worked Solution

$$8 = 2^3$$

$$12 = 2^2 \times 3$$

$$N = 2^a 3^b$$

$$a - 3 \text{ is even}$$

$$a - 2 \text{ is a multiple of 3}$$

$$a = 3, 5, 7, \dots \Rightarrow a = 5$$

$$b \text{ is even}$$

$$b - 1 \text{ is a multiple of 3}$$

$$b = 2, 4, 6, \dots \Rightarrow b = 4$$

$$N = 2^5 \times 3^4 = 2592$$

$$N/8 = 324 = 18^2$$

$$N/12 = 216 = 6^3$$

Extra primes can only increase the smallest N .

A square has even indices; a cube has multiples of 3.

The first candidate with $a - 2$ a multiple of 3 is 5.

Apply the same two conditions to the power of 3.

The exponent is positive because N is divisible by 12.

Use the smallest permitted indices.

Check both quotients.

4 Powers and integer reasoning

★★★★

Q19 Find the smallest positive integer k for which $180k$ is a square.

★★★

Q20 Find the smallest positive integer k for which $72k$ is a cube.

★★★

Q21 Find the smallest positive integer k such that $360/k$ is a square number.

★★★

Q22 $N = 2^a 3^b$, where a and b are non-negative integers. Its HCF with 72 is 12 and its LCM with 72 is 360. Find a and b , or explain why this is impossible.

Q23 Find all pairs of positive integers with HCF 6 and LCM 72. Count each pair only once.

Q24 Find the smallest positive integer N such that $N/12$ is a square number and $N/18$ is a cube number.

Solving principles

1. Choose the quantity

Largest equal groups suggest HCF; the first shared repeat suggests LCM.

2. Keep prime factors

Prime-factor form makes divisibility and perfect powers visible.

3. Find the smallest value

A smallest multiplier adds only the prime factors needed.

4. Check both conditions

A proposed pair must have the stated HCF and LCM, not just the right product.