

GCSE MATHS

Expanding and factorising

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

Contents

	Questions	Difficulty	Page
Topic recap			2
1. Single brackets and common factors	Q1–6	**	3
2. Double brackets and quadratic factors	Q7–12	**	5
3. Harder quadratics	Q13–18	***	7
4. Cubic products and structure	Q19–24	****	9
Solving principles			12

Topic recap

Distributive law

$a(b + c) = ab + ac$. Multiply every term inside the bracket, including its sign.

Common factors

Factorising reverses expansion. Take out the highest factor shared by every term.

Quadratic factors

For $x^2 + bx + c$, find two numbers with sum b and product c . For $ax^2 + bx + c$, split the middle term using product ac .

Difference of squares

$u^2 - v^2 = (u - v)(u + v)$. The middle terms cancel when the two brackets are expanded.

1. Single brackets and common factors

Example Problem 1

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Factorise fully $18x^2 - 12x$.

The idea

1. Divide each term by the greatest common factor to form the bracket.

Worked Solution

$$18x^2 - 12x = 6x(3x - 2)$$

The greatest common factor is $6x$.

$$6x(3x) - 6x(2) = 18x^2 - 12x$$

Check by distributing.

1 Single brackets and common factors

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Q1 Expand and simplify $3(2x - 5)$.

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Q2 Expand and simplify $-4(3x + 2)$.

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Q3 Expand and simplify $5x(2x - 3)$.

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Q4 Factorise fully $12x + 18$.

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Q5 Factorise fully $15x^2 - 10x$.

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Q6 Factorise fully $8x^3 + 12x^2$.

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2. Double brackets and quadratic factors

Example Problem 2

★ ★

Factorise $x^2 + 2x - 15$.

The idea

1. The middle term is the sum of the two bracket constants.

Worked Solution

$$5 + (-3) = 2$$

Find the sum and product.

$$5(-3) = -15$$

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

Use those constants.

2 Double brackets and quadratic factors

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Q7 Expand and simplify $(x + 3)(x + 5)$.

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Q8 Expand and simplify $(x - 4)(x + 7)$.

**

Q9 Expand and simplify $(x - 6)(x - 2)$.

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Q10 Factorise $x^2 + 11x + 28$.

**

Q11 Factorise $x^2 - 3x - 10$.

**

Q12 Factorise $x^2 - 11x + 24$.

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3. Harder quadratics

Example Problem 3

Factorise $6x^2 + 11x + 3$.

The idea

1. Split the middle term so the two groups share a bracket.

Worked Solution

$$ac = 18$$

Find two numbers with product 18 and sum 11.

$$9 + 2 = 11$$

$$6x^2 + 9x + 2x + 3$$

Split the middle term.

$$3x(2x + 3) + (2x + 3)$$

Factor each pair.

$$(3x + 1)(2x + 3)$$

Take out the common bracket.

3 Harder quadratics

Q13 Expand and simplify $(2x + 3)(x + 4)$.

**

Q14 Expand and simplify $(3x - 2)(2x + 5)$.

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Q15 Expand and simplify $(2x - 1)^2$.

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Q16 Factorise fully $2x^2 + 7x + 3$.

Q17 Factorise fully $6x^2 - x - 2$.

Q18 Factorise fully $9x^2 - 24x + 16$.

4. Cubic products and structure

Example Problem 4

Expand $(x - 1)(x + 2)(x + 4)$.

The idea

1. Complete one multiplication before starting the next.

Worked Solution

$$(x - 1)(x + 2) = x^2 + x - 2$$

Expand the first two brackets.

$$(x^2 + x - 2)(x + 4)$$

Multiply this result by the last bracket.

$$x^3 + 4x^2 + x^2 + 4x - 2x - 8$$

Multiply each term by x and by 4.

$$x^3 + 5x^2 + 2x - 8$$

Collect like terms.

4 Cubic products and structure

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Q19 Expand and simplify $(x + 1)(x + 2)(x + 3)$.

★★★

Q20 Expand and simplify $(2x - 1)(x + 2)(x - 3)$.

★★★

Q21 Factorise fully $3x^3 - 12x$.

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Q22 Factorise fully $(x + 3)^2 - 25$.

Q23 $x^2 + kx + 24 = (x + a)(x + b)$, where a and b are positive integers. Find all possible values of k .

Q24 Find p and q such that $(2x + p)(x + q) = 2x^2 + 11x + 12$, where p and q are positive integers.

Solving principles

1. Count the products

For double brackets there are four products before collecting terms.

2. Check by expanding

Your factors must reproduce every original coefficient.

3. Factorise fully

After removing a common factor, inspect the bracket for further factors.

4. Look for structure

A repeated bracket or a square can be treated as a single expression.