

GCSE MATHS

Direct and inverse proportion

Worked Solutions

Q1

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$$y = kx$$

$$k = \frac{18}{4} = \frac{9}{2}$$

$$y = \frac{9}{2}(10) = 45$$

Find the constant.

Use the new x value.

Q2

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$$y = \frac{7}{2}x$$

$$63 = \frac{7}{2}x$$

$$x = 63 \times \frac{2}{7} = 18$$

The constant is $28/8$.Substitute the given y .

Divide by the coefficient.

Q3

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$$8.75 \div 5 = 1.75$$

$$14 \times 1.75 = 24.50$$

Find the price of one notebook.

Multiply by the number required.

Q4

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$$e = kF$$

$$k = \frac{7.5}{12} = 0.625$$

$$11.25 = 0.625F$$

$$F = 18 \text{ N}$$

Find extension per newton.

Substitute the new extension.

Divide by 0.625.

Q5

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$$(a) \quad k = \frac{12}{15} = \frac{4}{5}$$

$$p = \frac{4}{5}q$$

$$(b) \quad p = \frac{4}{5} \times \frac{5}{2} = 2$$

Find the constant.

Write the relationship.

Substitute the fractional input.

Q6

$$1.15 \times 0.8 = 0.92$$

$$\frac{y_{\text{new}}}{y_{\text{old}}} = 0.92$$

$$1 - 0.92 = 0.08 = 8\%$$

Find the combined x multiplier.Direct proportion gives y the same multiplier.

Find the decrease.

Q7

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$$y = \frac{k}{x}$$

$$k = 3(20) = 60$$

$$y = \frac{60}{12} = 5$$

Find the constant product.

Divide the constant by the new x .**Q8**

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$$xy = 72$$

$$6x = 72$$

$$x = 12$$

Use the known pair.

Substitute the new y .

Divide by six.

Q9

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$$nt = 6(10) = 60$$

$$t = \frac{60}{15} = 4$$

The work requires 60 pump-hours.

Divide by the number of pumps.

Q10

**

$$d = 72(2.5) = 180 \text{ km}$$

$$t = \frac{180}{60} = 3 \text{ h}$$

Find the fixed distance.

Divide distance by the new speed.

Q11

$$\frac{y_{\text{new}}}{y_{\text{old}}} = \frac{1}{1.25} = 0.8$$

$$1 - 0.8 = 0.2 = 20\%$$

Invert the x multiplier.

Find the fraction lost.

Q12

$$8(15) = 120$$

$$120 - 8(5) = 80$$

$$t = \frac{80}{12} = \frac{20}{3}$$

Measure the whole job in worker-days.

Subtract the work already done.

Share the remaining work among 12 workers.

Q13

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$$y = kx^2$$

$$k = \frac{45}{9} = 5$$

$$y = 5(7^2) = 245$$

Square x before finding k .

Use the new input.

Q14

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$$y = k\sqrt{x}$$

$$k = \frac{10}{4} = \frac{5}{2}$$

$$y = \frac{5}{2}\sqrt{100} = 25$$

Use the square root of 16.

Substitute $x = 100$.

Q15

$$y = \frac{k}{x^2}$$

$$k = 18(4) = 72$$

$$8 = \frac{72}{x^2}$$

$$x^2 = 9$$

$$x = 3$$

Find the constant.

Substitute $y = 8$.

Use the positive square root.

Q16

$$V = 5r^3$$

$$r^3 = \frac{135}{5} = 27$$

$$r = 3$$

The constant is $40/8$.

Divide by five.

Take the cube root.

Q17

$$y = \frac{k}{\sqrt{x}}$$

$$k = 8(3) = 24$$

$$y = \frac{24}{8} = 3$$

Multiply by the square root to find k .

Use the square root of 64.

Q18

$$f^2 = 1.44$$

$$f = \sqrt{1.44} = 1.2$$

$$1.2 - 1 = 0.2 = 20\%$$

Let f be the positive x multiplier.

Take the square root.

Convert the increase factor to a percentage.

Q19

$$z = \frac{kx}{y}$$

$$9 = \frac{6k}{4}$$

$$k = 6$$

$$z = \frac{6(10)}{5} = 12$$

Combine the two relationships.

Find the constant.

Substitute the new pair.

Q20

$$F = \frac{km}{r^2}$$

$$15 = \frac{12k}{4}$$

$$k = 5$$

$$F = \frac{5(20)}{25} = 4$$

Use the known values.

Solve for the constant.

Use the new values.

Q21

$$\frac{z_{\text{new}}}{z_{\text{old}}} = \frac{1.1^2}{0.8}$$

$$\frac{1.21}{0.8} = 1.5125$$

51.25% increase

Square the x multiplier and divide by the y multiplier.

Calculate the combined multiplier.

Subtract one and convert to a percentage.

Q22

$$z \propto \frac{1}{x^2}$$

$$\frac{z_{\text{new}}}{18} = \left(\frac{2}{6}\right)^2$$

$$z = 2$$

Substitute the first relationship into the second.

The inverse-square scale factor is one ninth.

Multiply 18 by one ninth.

Q23

$$xy = 60$$

$$y = x + 11$$

$$x(x + 11) = 60$$

$$(x + 15)(x - 4) = 0$$

$$x = 4$$

$$y = 15$$

Write the constant product and difference.

Substitute for y .

Expand, rearrange and factorise.

Reject the negative root.

Q24

$$\frac{240}{6(5)} = 8$$

$$240 - 6(2)(8) = 144$$

$$4(8) = 32$$

$$t = \frac{144}{32} = 4.5 \text{ h}$$

Each machine produces eight items per hour.

Find the number of items still needed.

Four machines produce 32 items per hour.

Divide the remaining items by the new rate.