

GCSE MATHS

Direct and inverse proportion

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Direct proportion

$y \propto x$ means $y = kx$. Find k from a known pair; the graph passes through the origin.

Inverse proportion

$y \propto \frac{1}{x}$ means $y = \frac{k}{x}$. The product xy is constant.

Powers matter

$y \propto x^2$ means $y = kx^2$. Doubling x multiplies y by four.

Combined proportion

If z is directly proportional to x and inversely proportional to y , write $z = \frac{kx}{y}$.

1. Direct proportion

Example Problem 1

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$y \propto x$. When $x = 6$, $y = 21$. Find y when $x = 14$.

The idea

1. Find y divided by x using the known pair.
2. Use that same constant with the new x .

Worked Solution

$$y = kx$$

Introduce the constant.

$$21 = 6k$$

Use the known pair.

$$k = \frac{7}{2}$$

$$y = \frac{7}{2}(14) = 49$$

Substitute the required x .

1 Direct proportion

Q1 y is directly proportional to x . When $x = 4$, $y = 18$. Find y when $x = 10$.

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Q2 $y \propto x$. When $x = 8$, $y = 28$. Find x when $y = 63$.

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Q3 Five identical notebooks cost £8.75. Find the cost of 14 notebooks at the same unit price.

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Q4 A spring extension is directly proportional to the applied force. A force of 12 N produces 7.5 cm extension. Find the force needed for 11.25 cm extension.

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Q5 $p \propto q$. When $q = 15$, $p = 12$. (a) Express p in terms of q . (b) Find p when $q = \frac{5}{2}$.

**

Q6 y is directly proportional to x . The value of x increases by 15% and then decreases by 20%. Find the overall percentage change in y .

2. Inverse proportion

Example Problem 2

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Nine identical printers finish a job in 8 hours. How long would 12 printers take at the same constant rate?

The idea

1. Keep the amount of work fixed.
2. Use the constant product of printer count and time.

Worked Solution

$$nt = 9(8) = 72$$

$$12t = 72$$

$$t = 6 \text{ h}$$

The job requires 72 printer-hours.

The same work is shared between 12 printers.

Divide by 12.

2 Inverse proportion

Q7 y is inversely proportional to x . When $x = 3$, $y = 20$. Find y when $x = 12$.

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Q8 $y \propto \frac{1}{x}$. When $x = 8$, $y = 9$. Find x when $y = 6$.

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Q9 Six identical pumps empty a tank in 10 hours. How long would 15 such pumps take, assuming their rates add and remain constant?

**

Q10 A fixed journey takes 2.5 hours at 72 km/h. How long does it take at 60 km/h?

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Q11 y is inversely proportional to x . The value of x increases by 25%. Find the percentage change in y .

Q12 Eight workers can complete a job in 15 days. They work for 5 days, then four more equally productive workers join. How many further days are needed?

3. Powers and roots

Example Problem 3

$y \propto x^2$. When $x = 4$, $y = 48$. Find the positive x when $y = 147$.

The idea

1. Find k in $y = kx^2$.
2. Solve for x^2 , then take the root.

Worked Solution

$$y = kx^2$$

$$48 = 16k$$

$$k = 3$$

$$147 = 3x^2$$

$$x^2 = 49$$

$$x = 7$$

Square the known x before substituting.

Use the constant with the new y .

Divide by three and take the positive root.

3 Powers and roots

Q13 $y \propto x^2$. When $x = 3$, $y = 45$. Find y when $x = 7$.

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Q14 $y \propto \sqrt{x}$. When $x = 16$, $y = 10$. Find y when $x = 100$.

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Q15 $y \propto \frac{1}{x^2}$. When $x = 2$, $y = 18$. Find the positive x when $y = 8$.

Q16 $V \propto r^3$. When $r = 2$, $V = 40$. Find r when $V = 135$, taking $r > 0$.

Q17 $y \propto \frac{1}{\sqrt{x}}$. When $x = 9$, $y = 8$. Find y when $x = 64$.

Q18 $y \propto x^2$, where $x > 0$. The value of y increases by 44%. Find the percentage increase in x .

4. Combining relationships

Example Problem 4

$z \propto \frac{x}{y^2}$. When $x = 8$, $y = 2$, $z = 10$. Find z when $x = 18$, $y = 3$.

The idea

1. Put the squared variable in the denominator.
2. Substitute the given values of x , y and z to find k , then use the new values.

Worked Solution

$$z = \frac{kx}{y^2}$$

Write the combined relationship.

$$10 = \frac{8k}{4}$$

Use the known values.

$$k = 5$$

$$z = \frac{5(18)}{9} = 10$$

Substitute the new x and y .

4 Combining relationships

Q19 z is directly proportional to x and inversely proportional to y . When $x = 6$, $y = 4$, $z = 9$. Find z when $x = 10$, $y = 5$.

Q20 $F \propto \frac{m}{r^2}$. When $m = 12$, $r = 2$, $F = 15$. Find F when $m = 20$, $r = 5$.

Q21 $z \propto \frac{x^2}{y}$. The value of x rises by 10% and y falls by 20%. Find the percentage change in z .

Q22 $y \propto x^2$ and $z \propto \frac{1}{y}$. When $x = 2$, $z = 18$. Find z when $x = 6$.

Q23 $y \propto \frac{1}{x}$ and $y = 12$ when $x = 5$. Both x and y are positive. Find the pair for which $y - x = 11$.

Q24 The time t to produce a batch is proportional to the number of items N and inversely proportional to the number of machines m . Six machines make 240 items in 5 hours. After 2 hours, two machines stop. How much longer is needed to finish the batch?

Solving principles

1. Write an equation

Replace the proportion sign with an equals sign and a constant k .

2. Find k once

Use a known pair before substituting the new value.

3. Use scale factors

If $y = kx^2$ and x is multiplied by f , then y is multiplied by f^2 .

4. Apply the model conditions

Keep units consistent and use positive roots when quantities are lengths, times or masses.