

GCSE MATHS

Cumulative frequency and box plots

Worked Solutions

Q1

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$$\begin{aligned} & 6 \\ 6 + 14 &= 20 \\ 20 + 18 &= 38 \\ 38 + 12 &= 50 \end{aligned}$$

Start the running total.

Continue adding each class.

Q2

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$$\begin{aligned} f_1 &= 9 \\ 25 - 9 &= 16 \\ 42 - 25 &= 17 \\ 60 - 42 &= 18 \end{aligned}$$

The first cumulative count is the first frequency.

Subtract consecutive cumulative counts.

Q3

★★

$$\begin{aligned} N/4 &= 20 \\ N/2 &= 40 \\ 3N/4 &= 60 \end{aligned}$$

Use quarter, half and three-quarter positions.

Q4

★★

$$f = 50 - 12 = 38$$

Subtract the count up to 10 from the count up to 30.

Q5

★★

$$\begin{aligned} f_{>20} &= 100 - 64 = 36 \\ 36/100 \times 100 &= 36\% \end{aligned}$$

Take the complement.

Convert to a percentage.

Q6

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CF = 10, 40, 70, 80

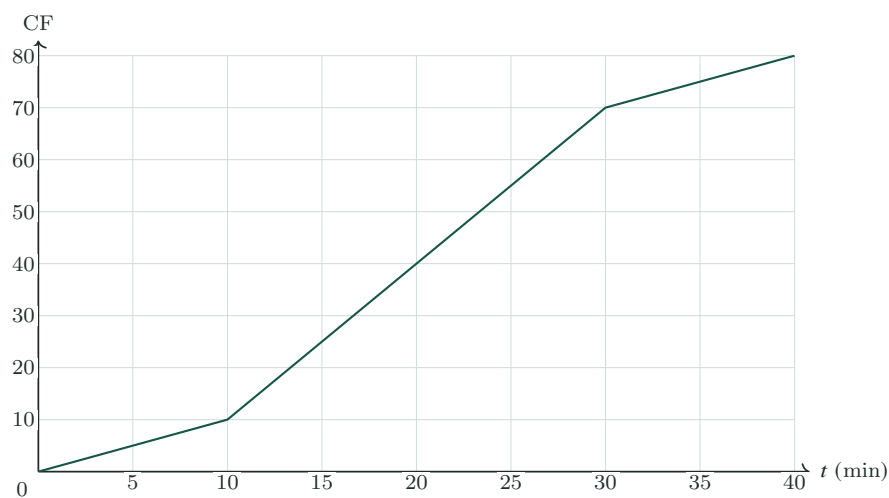
(0, 0), (10, 10), (20, 40)

(30, 70), (40, 80)

Add the frequencies successively.

Plot zero at the first lower boundary and cumulative counts at upper boundaries.

Complete the points and join successive points as requested.



Q7

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CF = 40

 $t \approx 17.5$ minutes

Find the required cumulative-frequency position.

Read across to the graph, then down to the time axis.

Q8

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 $60 - 48 = 12$

Subtract the cumulative count at 30 from the total.

Q9

CF = 25

 $t \approx 12.5$ minutes

Find the required cumulative-frequency position.

Read across to the graph, then down to the time axis.

Q10

 $t \approx 29$ minutes

Read the estimate from the graph.

Q11

$$Q_1 = 10$$

$$Q_3 = 25$$

$$\text{IQR} = 25 - 10 = 15.0 \text{ min}$$

Read the times at cumulative frequencies 20 and 60.

Subtract the lower quartile from the upper quartile.

Q12

$$t \approx 43 \text{ minutes}$$

Read the estimate from the graph.

Q13

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$$\text{median} = 16 \text{ min}$$

Read the line inside the box.

Q14

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$$20 - 8 = 12 \text{ min}$$

Subtract the lower quartile from the upper quartile.

Q15

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$$40 - 6 = 34 \text{ min}$$

Subtract the minimum from the maximum.

Q16

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$$75\% - 25\% = 50\%$$

The box runs from the lower to the upper quartile.

Q17

$$Q_3 = 28 \text{ minutes}$$

The upper quartile leaves one quarter above it.

$$\text{estimate} = 120/4 = 30$$

Use a quarter of the total.

Q18

(a) $15 + 5 = 20 \text{ min}$

The median increases by five minutes.

(b) $24 - 9 = 15 \text{ min}$

Adding the same amount to every value leaves the interquartile range unchanged.

Q19

$18 < 22$

Group A has the lower median.

$5 < 8$

 B has the smaller interquartile range.

Q20

(a) $\text{median}_{\text{new}} = 2(18) = 36$

Positive scaling preserves the order.

(b) $\text{IQR}_{\text{new}} = 2(24 - 12) = 24$

Scale the spread by the same factor.

Q21

$$\bar{x} = \frac{\text{sum of all values}}{N}$$

The summary does not fix that sum.

The mean uses every observation.

Interior observations can change while the summary stays fixed.

Q22

$$f_A \approx 10$$

$$f_B \approx 20$$

$$\frac{10 + 20}{40 + 80} \times 100 = 25\%$$

Take a quarter of each group.

Combine counts and totals.

Q23

$$60/2 = 30$$

$$t \approx 16 \text{ minutes}$$

Find the halfway cumulative frequency.

Read across to the graph, then down to the time axis.

Q24

$$20 = 8a \Rightarrow a = 2.5$$

$$55 = 2.5(20) + b$$

$$b = 5$$

Both quartiles are multiplied by a and increased by b . Their difference is multiplied by a ; adding b to both leaves the difference unchanged.

Use the converted median.

Subtract fifty.