

GCSE MATHS

Cumulative frequency and box plots

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Cumulative counts

Cumulative frequency counts observations up to an upper boundary. Add frequencies as you move through the classes. Join successive plotted points with straight segments in these questions.

Quartile positions

For grouped data with total frequency N , use positions $N/4$, $N/2$ and $3N/4$ on a cumulative frequency graph. Read across to the graph, then down to the measurement axis.

Reading a box plot

A box plot shows the minimum, lower quartile, median, upper quartile and maximum. The box spans the middle half of the data.

Comparisons

The interquartile range is $Q_3 - Q_1$ and the range is maximum minus minimum. Compare typical values using medians and consistency using a suitable measure of spread.

A calculator may be used.

1. Cumulative frequency tables

Example Problem 1

*

The table shows journey times. Complete a cumulative frequency column.

Time t (minutes)	Frequency	Cumulative frequency
$0 < t \leq 10$	5	
$10 < t \leq 20$	12	
$20 < t \leq 30$	8	
$30 < t \leq 40$	15	

The idea

1. Add frequencies from the first class onward.
2. The final cumulative count must equal the total.

Worked Solution

$$\begin{aligned} & 5 \\ 5 + 12 &= 17 \\ 17 + 8 &= 25 \\ 25 + 15 &= 40 \end{aligned}$$

Begin the running total.

Add the next classes.

1. Cumulative frequency tables

Example Problem 2

**

The table shows some times. Draw a cumulative frequency graph, joining successive points with straight segments.

Time t (minutes)	Frequency
$0 < t \leq 10$	5
$10 < t \leq 20$	15
$20 < t \leq 30$	30
$30 < t \leq 40$	10

The idea

1. Add the class frequencies to form running totals.
2. Plot each total at its upper class boundary; begin at the first lower boundary with cumulative frequency zero.

Worked Solution

$$CF = 5, 20, 50, 60$$

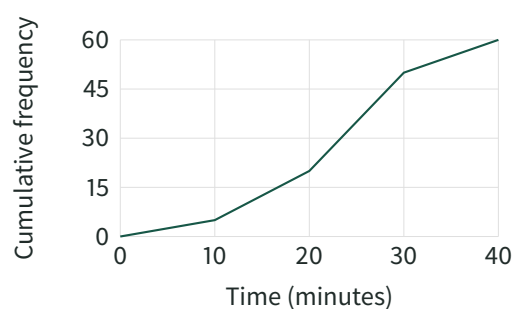
$$(0, 0), (10, 5), (20, 20)$$

$$(30, 50), (40, 60)$$

Add successive frequencies.

Use class boundaries as the horizontal coordinates.

Join with straight segments; check the graph ends at cumulative frequency 60.



1 Cumulative frequency tables

**

Q1 Complete the missing column in the table.

*

Time t (minutes)	Frequency	Cumulative frequency
$0 < t \leq 10$	6	
$10 < t \leq 20$	14	
$20 < t \leq 30$	18	
$30 < t \leq 40$	12	

Q2 Complete the missing column in the table.

*

Time t (minutes)	Frequency	Cumulative frequency
$0 < t \leq 10$		9
$10 < t \leq 20$		25
$20 < t \leq 30$		42
$30 < t \leq 40$		60

Q3 For a sample of 80 observations, state the cumulative frequency positions of Q_1 , the median and Q_3 .

**

Q4 The table summarises some times. How many observations satisfy $10 < t \leq 30$?

**

Time t (minutes)	Cumulative frequency
$t \leq 10$	12
$t \leq 20$	35
$t \leq 30$	50

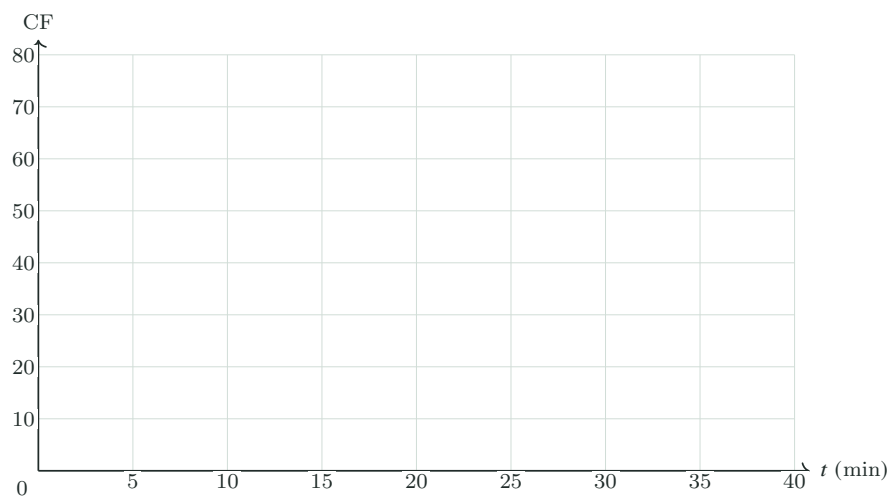
Q5 A sample has 100 observations. Its cumulative count at 20 is 64. What percentage have values greater than 20?

**

- Q6** The table shows some times. Draw a cumulative frequency graph, joining successive points with straight segments.

**

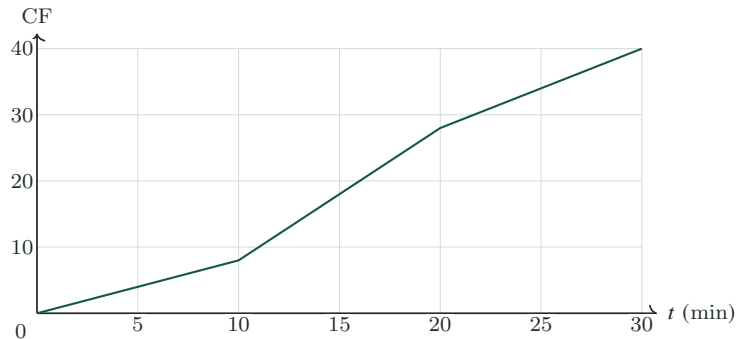
Time t (minutes)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	30
$20 < t \leq 30$	30
$30 < t \leq 40$	10



2. Quartiles and estimates

Example Problem 3

The cumulative frequency graph shows 40 journey times. Estimate the median time.



The idea

1. Find half the total frequency.
2. Read across from this cumulative frequency to the graph, then down to the time axis.

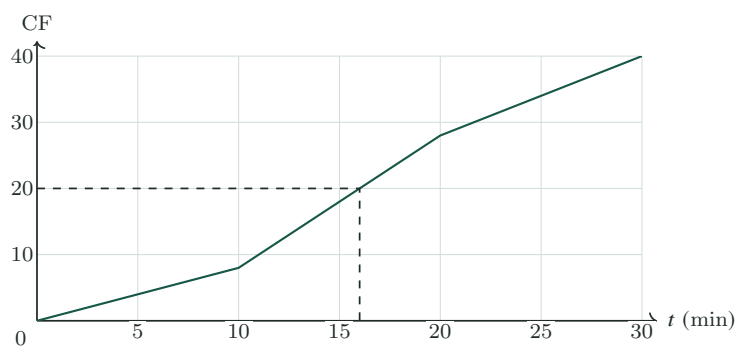
Worked Solution

$$40/2 = 20$$

Find the halfway cumulative frequency.

$$t \approx 16 \text{ minutes}$$

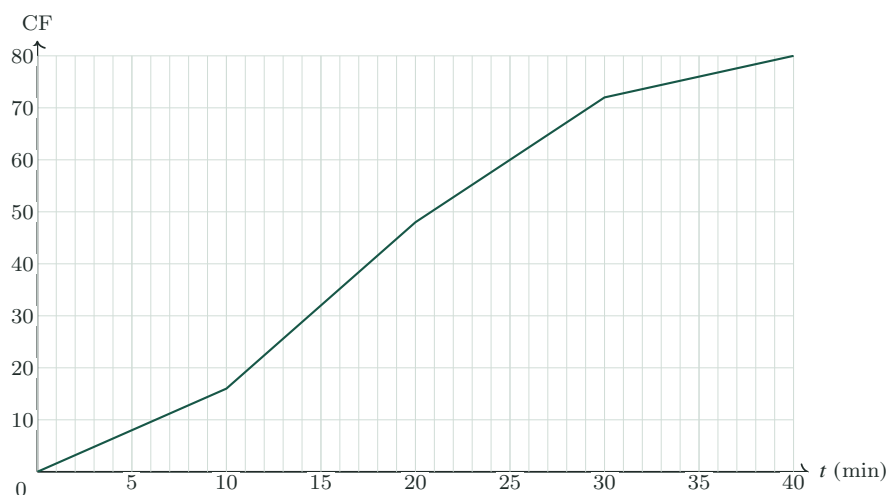
Read across from 20 to the graph, then down to the time axis.



2 Quartiles and estimates

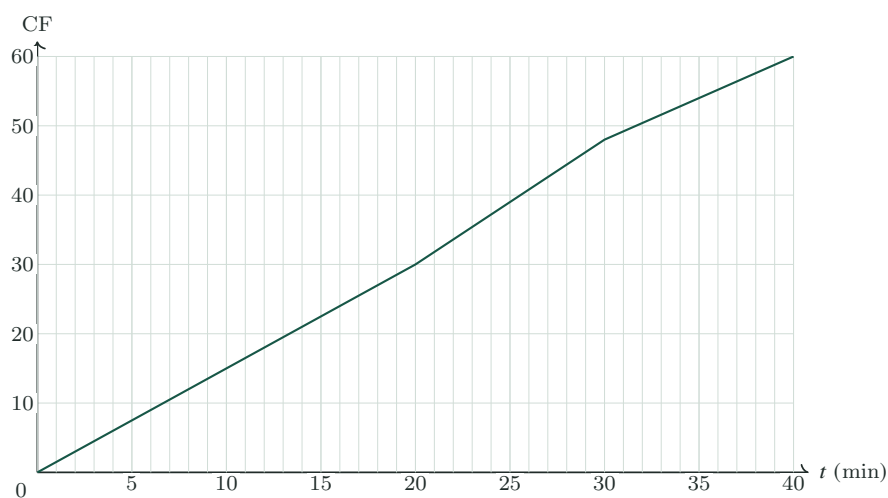
Q7 Use the cumulative frequency graph to estimate the median time.

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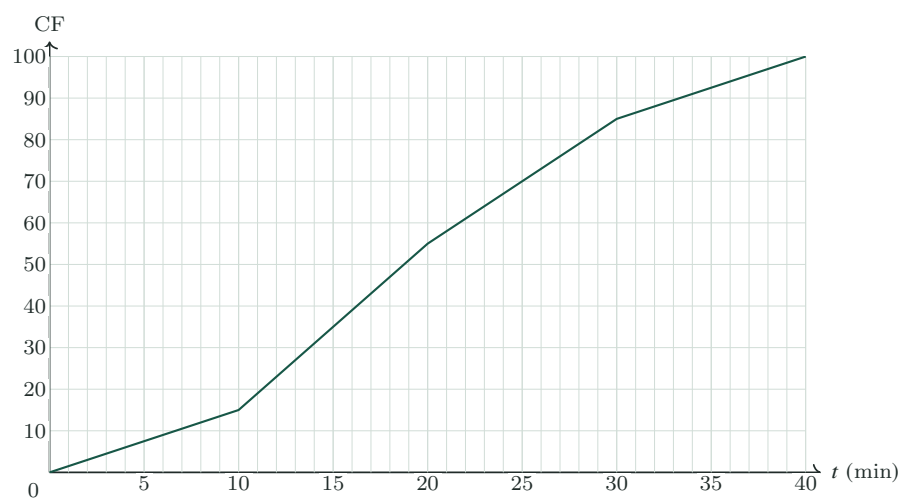


Q8 Use the graph to estimate how many observations have $t > 30$ minutes.

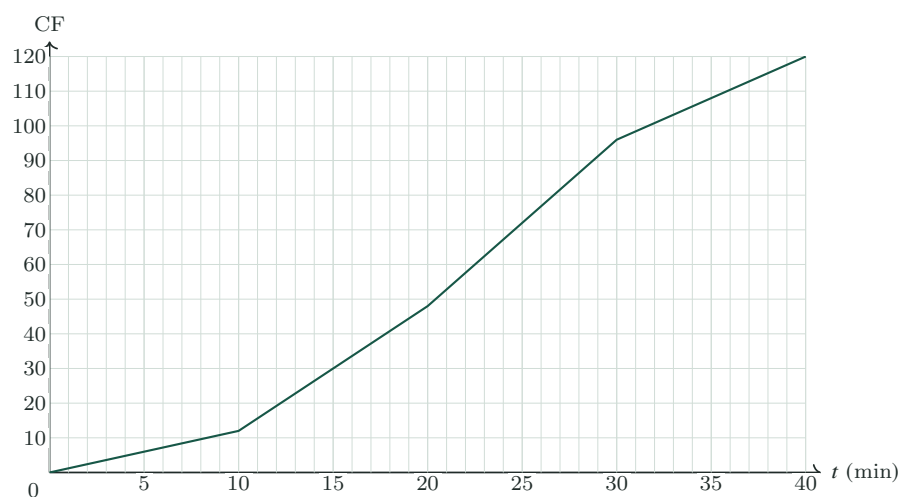
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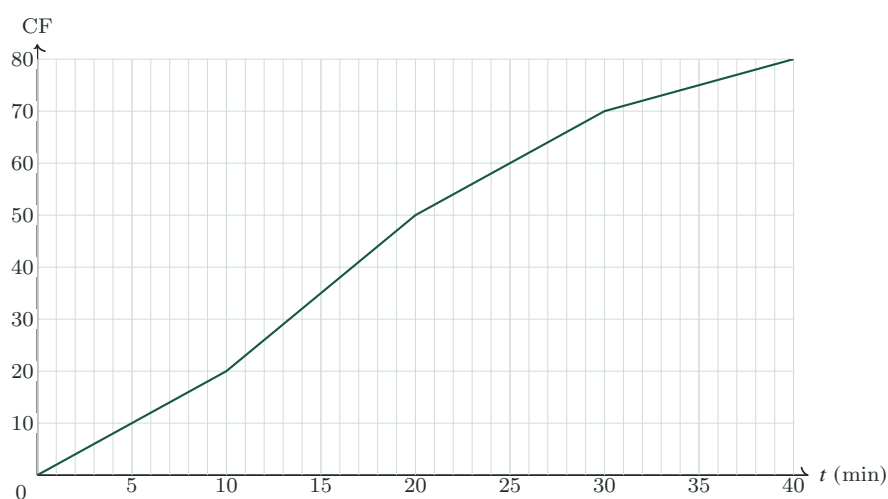
Q9 Estimate the lower quartile from the graph.



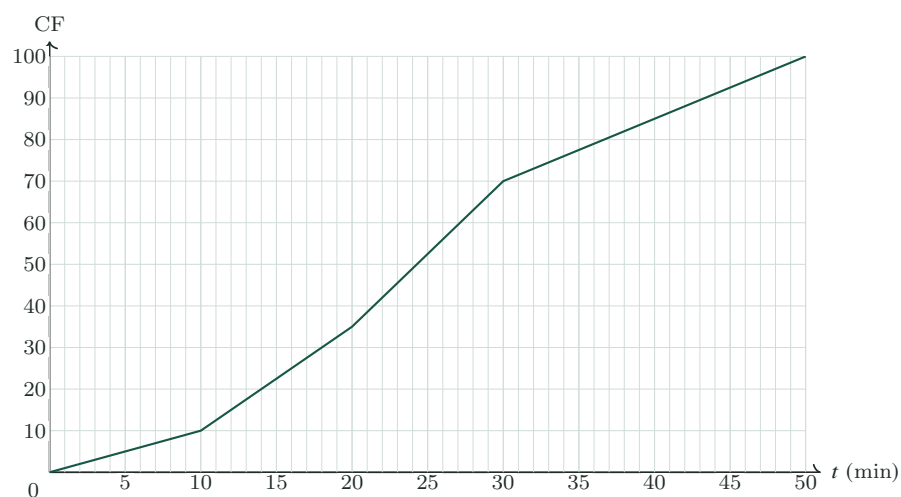
Q10 Estimate the upper quartile from the graph.



Q11 Estimate the interquartile range from the graph.



Q12 Estimate the time exceeded by 10% of observations.



3. Reading box plots

Example Problem 4

**

The box plot shows test scores. Find the median and IQR.



The idea

1. The whiskers show the overall range.
2. The box itself gives the quartiles.

Worked Solution

$$\text{median} = 18$$

Read the central line.

$$\text{IQR} = 26 - 12 = 14$$

Subtract the ends of the box.

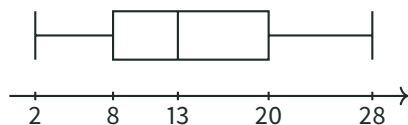
3 Reading box plots

Q13 The box plot shows completion times in minutes. State the median.

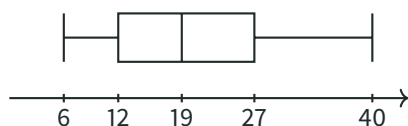
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**Q14** Find the interquartile range of the times shown.

**

**Q15** Find the range of the times shown.

**



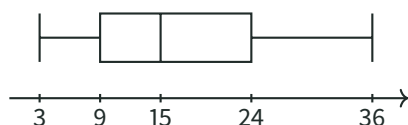
Q16 What percentage of the distribution is represented by the box between 15 and 32 minutes? **



Q17 The box plot summarises 120 observations. Estimate how many are above 28 minutes. ***



Q18 Every time in the box plot is increased by 5 minutes. Find (a) the new median; (b) the new interquartile range. ***



4. Comparisons and statistical reasoning

Example Problem 5

Class A has median score 65 and IQR 12; class B has median 61 and IQR 8. Compare the classes.

The idea

1. Use one statement about the median.
2. Use a separate statement about spread.

Worked Solution

$$65 > 61$$

Class A has the higher typical score.

$$8 < 12$$

B has the more consistent middle half of scores.

4 Comparisons and statistical reasoning

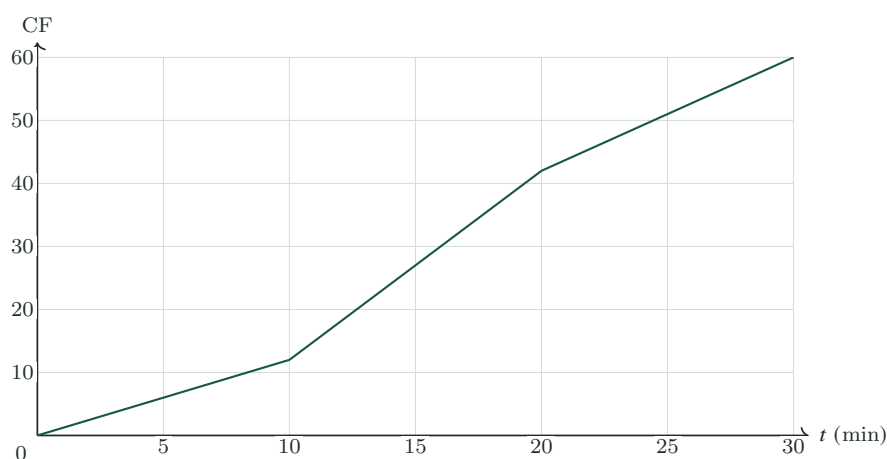
Q19 Group A has median journey time 18 minutes and IQR 8 minutes. Group B has median 22 minutes and IQR 5 minutes. Compare their typical times and consistency.

Q20 A set of scores has lower quartile 12, median 18 and upper quartile 24. Every score is doubled. Find (a) the new median; (b) the new IQR.

Q21 Two groups have the same minimum, lower quartile, median, upper quartile and maximum. Must their means be equal? Explain.

- Q22** Group A has 40 observations and group B has 80. In each group, about 25% exceed 30 minutes. Estimate the percentage exceeding 30 minutes in the combined group.

- Q23** The cumulative frequency graph shows 60 journey times. Estimate the median time.



- Q24** Each value x in a set of data is changed to $y = ax + b$, where $a > 0$. The original data have median 20 and interquartile range 8. The new data have median 55 and interquartile range 20. Find a and b .

Solving principles

1. Distinguish counts and values

A quartile position is a cumulative count; the quartile itself has the units of the measurements.

2. Read estimates from the graph

Use the straight section between the two plotted points when the required cumulative frequency lies between them.

3. Compare in context

Say which group tends to have longer times or more consistent scores, rather than just quoting two numbers.

4. Avoid conclusions from missing detail

A minimum, lower quartile, median, upper quartile and maximum does not determine the mean or every individual observation.