

GCSE MATHS

Coordinates and column vectors

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Coordinates

Write horizontal position first, then vertical position. The midpoint averages the two x -coordinates and the two y -coordinates.

Vectors between points

A vector gives a change in position, called a displacement. $\overrightarrow{AB}$ is B minus A , coordinate by coordinate.

Vector arithmetic

Add corresponding components. To multiply a vector by a number, multiply both components. A negative multiplier reverses the direction.

Parallel vectors

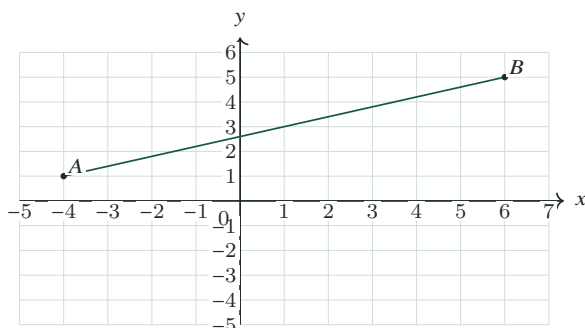
For two non-zero parallel vectors, one is a multiple of the other. Equal vectors have the same components, wherever they start.

1. Coordinates and midpoints

Example Problem 1

★ ★

A is $(-4, 1)$ and B is $(6, 5)$. Find the midpoint of AB .



The idea

1. Average each coordinate separately.

Worked Solution

$$M = \left(\frac{-4 + 6}{2}, \frac{1 + 5}{2} \right)$$

$$M = (1, 3)$$

Add the endpoints, then halve.

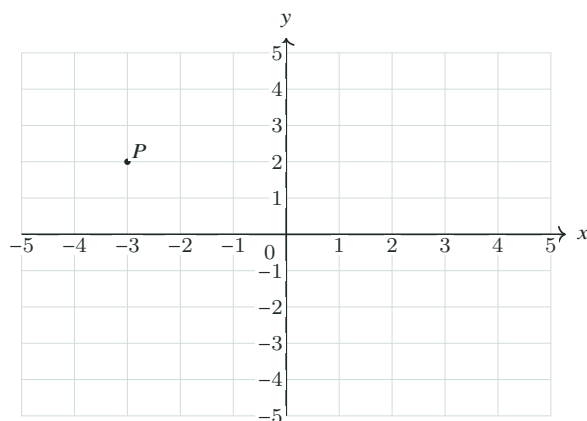
The midpoint lies halfway in both directions.

1 Coordinates and midpoints

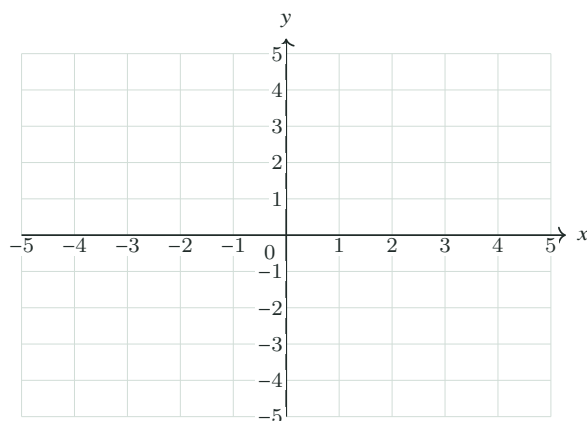
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Q1 Write the coordinates of P .

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**Q2** Plot $A = (2, -3)$ on the grid.

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**Q3** Find the midpoint of $(-6, 4)$ and $(2, -2)$.

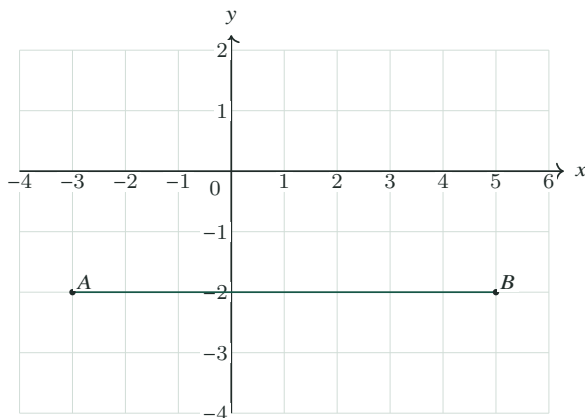
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Q4 A is $(1, 7)$. The midpoint of AB is $(4, 3)$. Find B .

**

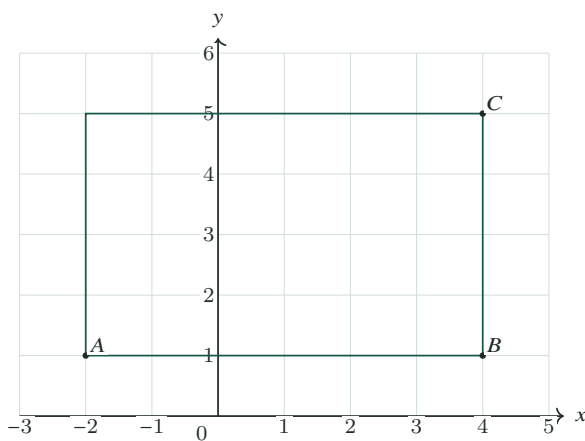
Q5 $A = (-3, -2)$ and $B = (5, -2)$. Find the length AB .

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Q6 $A = (-2, 1)$, $B = (4, 1)$ and $C = (4, 5)$ are three vertices of a rectangle with sides parallel to the axes. Find its area.

**



2. Column vectors

Example Problem 2

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A is $(2, -3)$ and B is $(-4, 5)$. Find the column vector $\overrightarrow{AB}$.

The idea

1. Subtract the coordinates of A from those of B .

Worked Solution

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} -4 - 2 \\ 5 - (-3) \end{pmatrix} \\ \overrightarrow{AB} &= \begin{pmatrix} -6 \\ 8 \end{pmatrix}\end{aligned}$$

Destination minus start.

Move left six and up eight.

2 Column vectors

Q7 $A = (1, 2)$ and $B = (5, -1)$. Find the column vector $\overrightarrow{AB}$.

**

Q8 $P = (-3, 4)$ and $Q = (2, -2)$. Find the column vector PQ .

**

Q9 $B = (6, 1)$ and $A = (2, 5)$. Find the column vector BA .

**

Q10 $A = (3, -2)$ and $\overrightarrow{AB} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$. Find B .

**

Q11 $B = (6, 1)$ and $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. Find A .

**

Q12 A translation maps $(-2, 5)$ to $(4, -1)$. Where does it map $(3, 2)$?

3. Combining vectors

Example Problem 3

Given $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find $2\mathbf{a} - \mathbf{b}$.

The idea

1. Multiply $\mathbf{a}$ by 2, then subtract each component of $\mathbf{b}$.

Worked Solution

$$2\mathbf{a} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$$2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 4 - (-3) \\ -2 - 4 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \end{pmatrix}$$

Double both components.

Subtract carefully when a component is negative.

3 Combining vectors

Q13 $a = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $b = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. Find $a + b$.

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Q14 $a = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Find $a - b$.

**

Q15 Given $a = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, find $3a$.

**

Q16 $a = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Find $2a - 3b$.

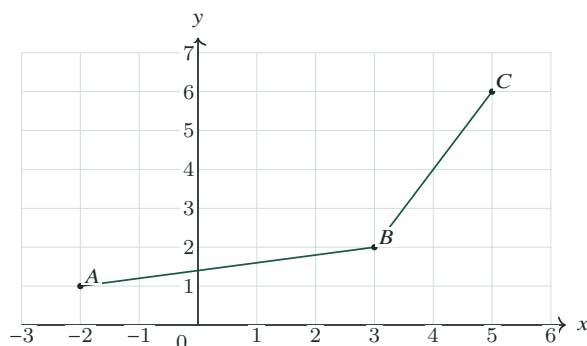
Q17 $a = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Find $-a + 2b$.

Q18 $a = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $b = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$. Find $3a + b$.

4. Coordinate reasoning

Example Problem 4

A , B , C and D are consecutive vertices of a parallelogram. $A = (-2, 1)$, $B = (3, 2)$, $C = (5, 6)$. Find D .



The idea

1. Opposite sides have equal directed displacement: AD equals BC .

Worked Solution

$$\overrightarrow{BC} = \begin{pmatrix} 5 - 3 \\ 6 - 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Find the movement from B to C .

$$D = (-2 + 2, 1 + 4) = (0, 5)$$

Apply the same movement to A .

4. Coordinate reasoning

Example Problem 5

$$p \begin{pmatrix} 1 \\ 2 \end{pmatrix} + q \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \end{pmatrix}. \text{ Find } p \text{ and } q.$$

The idea

1. Write one equation from each component and eliminate a variable.

Worked Solution

$$p + 2q = 8$$

Compare horizontal and vertical components.

$$2p - q = 1$$

$$4p - 2q = 2$$

Double the second equation.

$$5p = 10 \Rightarrow p = 2$$

Add the equations.

$$2 + 2q = 8 \Rightarrow q = 3$$

Substitute to find q .

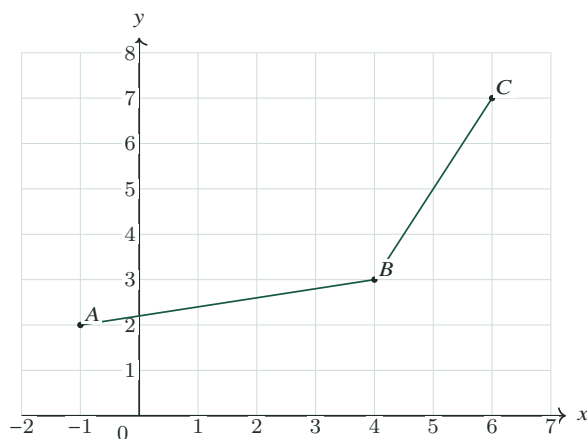
4 Coordinate reasoning

Q19 Find k if $\begin{pmatrix} 6 \\ -9 \end{pmatrix} = k \begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

Q20 Find t so that $\begin{pmatrix} 4 \\ t \end{pmatrix}$ is parallel to $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$.

Q21 $A = (1, 2)$, $B = (7, 5)$. P lies one third of the way from A to B . Find P .

- Q22** $A = (-1, 2)$, $B = (4, 3)$, $C = (6, 7)$. These are consecutive vertices of parallelogram $ABCD$. Find D . ***



- Q23** $A = (0, 1)$, $B = (2, 5)$ and $C = (5, 11)$. Show that the three points lie on one straight line. ***

- Q24** $a = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Find p and q if $pa + qb = \begin{pmatrix} 1 \\ 11 \end{pmatrix}$. ***

Solving principles

1. Draw the movement

Use a diagram to check signs and direction.

2. Subtract in order

Destination minus start gives the displacement.

3. Use components

A vector equality gives one horizontal and one vertical equation.

4. Check the geometry

Midpoints, parallel sides and displacement sums can verify a coordinate result.