

GCSE MATHS

Constructions and loci

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Constructions

Use a ruler and compasses, keeping the construction arcs visible. A perpendicular bisector passes through the midpoint of a line segment at 90° .

Angle bisectors

An angle bisector divides an angle into two equal angles. Points on it are equally far from the two sides, measured perpendicularly.

Loci

A locus is the set of all points satisfying a condition. A fixed distance from a point gives a circle; equal distance from two points gives their perpendicular bisector.

Regions

Within a distance gives the inside of a boundary. Use the overlap when both conditions must hold. A strict inequality excludes its boundary; an inclusive one includes it.

Use a ruler and compasses for construction questions. Print at 100% size for distances in centimetres.

1. Perpendicular bisectors

Example Problem 1

**

Construct the perpendicular bisector of AB .



The idea

1. Draw equal arcs from the endpoints and join their intersections.

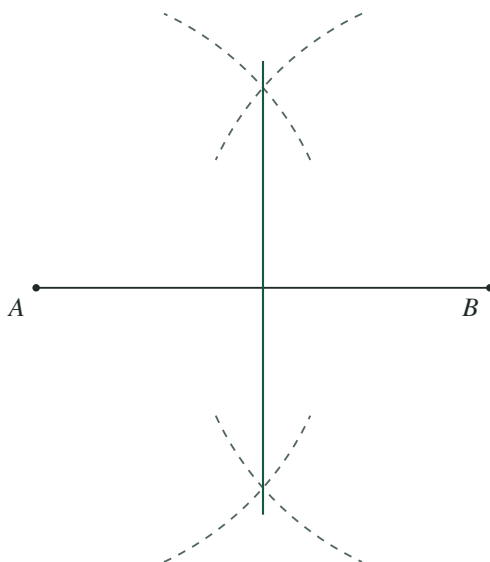
Worked Solution

Use the same radius greater than half AB

The arcs must cross above and below.

Join the two intersections

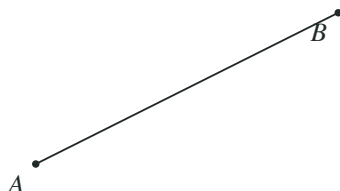
Their equal distances from A and B establish the bisector.



1 Perpendicular bisectors

Q1 Construct the perpendicular bisector of AB . Leave your arcs visible.

*

**Q2** P is on the perpendicular bisector of AB . $PA = 5$ cm. Find PB .

**

Q3 Construct the perpendicular bisector of AB . Leave your arcs visible.

**



- Q4** Use a ruler and compasses to divide AB into four equal parts. Leave your construction arcs visible. **



- Q5** A and B are shown. Construct the locus of points equally far from A and B . ***

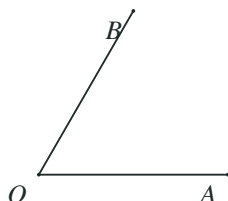


- Q6** Two circles of equal radius have centres A and B and intersect at P and Q . Explain why the line PQ is the perpendicular bisector of AB . ***

2. Angles and perpendiculars

Example Problem 2

Construct the bisector of angle AOB .



The idea

1. Mark the sides with one arc from O , then use equal arcs from the two marks.

Worked Solution

Arc from O meets both sides

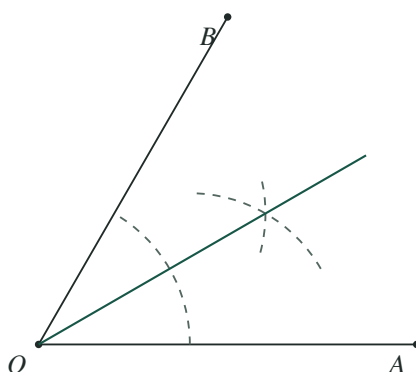
The two marks are equally far from O .

Equal arcs cross inside the angle

Their crossing is equally far from the two marks.

Join O to that crossing

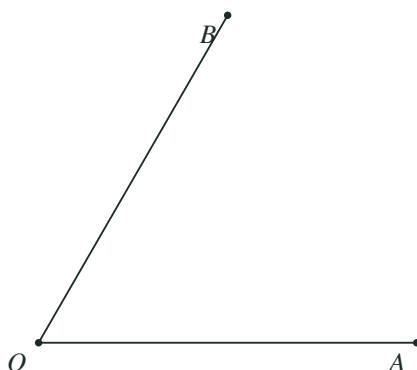
The two resulting triangles are congruent.



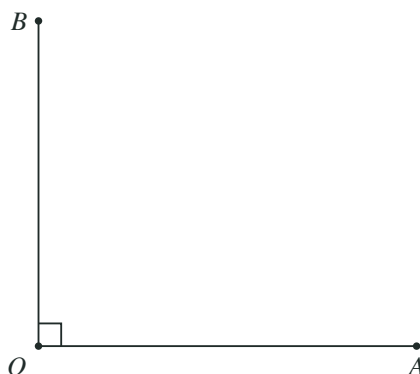
2 Angles and perpendiculars

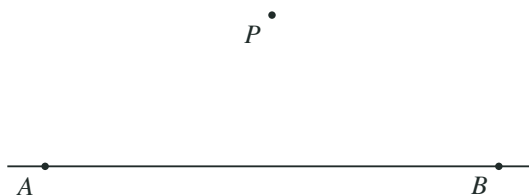
Q7 Construct the bisector of angle AOB . Leave your arcs visible.

**

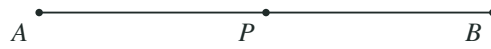
**Q8** Angle AOB is a right angle. Use a ruler and compasses to construct a ray OP inside it so that angle $AOP = 22.5^\circ$. Leave your construction arcs visible.

**

**Q9** Construct the perpendicular from P to line AB . Leave your arcs visible.



Q10 Construct a perpendicular to AB at P , where P lies on AB .



Q11 Using the segment AB , construct an equilateral triangle above AB .



Q12 Construct a 30° angle above OA at O . Leave your arcs visible.



3. Single loci

Example Problem 3

**

Draw all points 2 cm from O .

O •

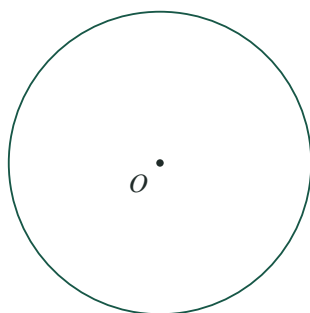
The idea

1. Set the compass radius to the required distance and draw a complete circle.

Worked Solution

$$OP = 2 \text{ cm}$$

Every point on the circle has the required distance.



3 Single loci

Q13 Construct the locus of points 3 cm from O .

*

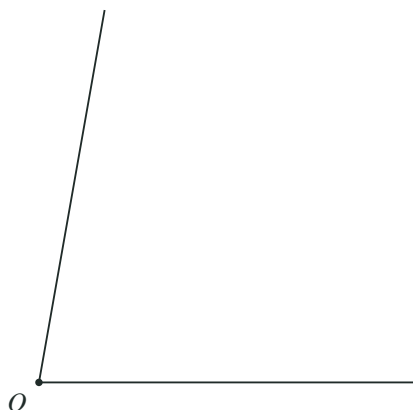
 O •**Q14** Construct the locus of points 2 cm from line l .

**

 l **Q15** Why are there two lines in the locus of points 2 cm from a straight line?

**

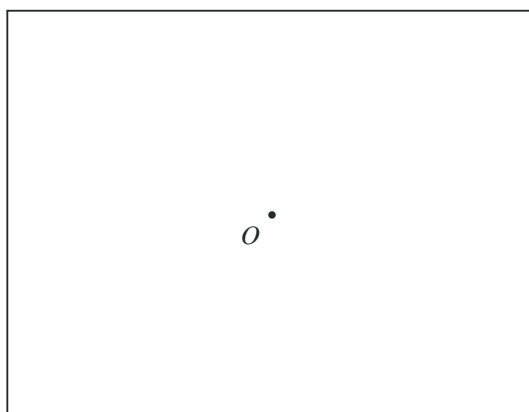
- Q16** Construct the locus of points inside the angle that are equally far from both sides. Leave your arcs visible. **



- Q17** A point P is 3 cm from O . O is 5 cm from a straight line. What is the least possible distance from P to that line? ***



- Q18** Shade all points inside the box shown that are at most 2 cm from O . Include the boundary. ***



4. Combining conditions

Example Problem 4

A and B are 6 cm apart. Construct points 4 cm from both A and B .



The idea

1. Draw one circle for each condition; their intersections satisfy both.

Worked Solution

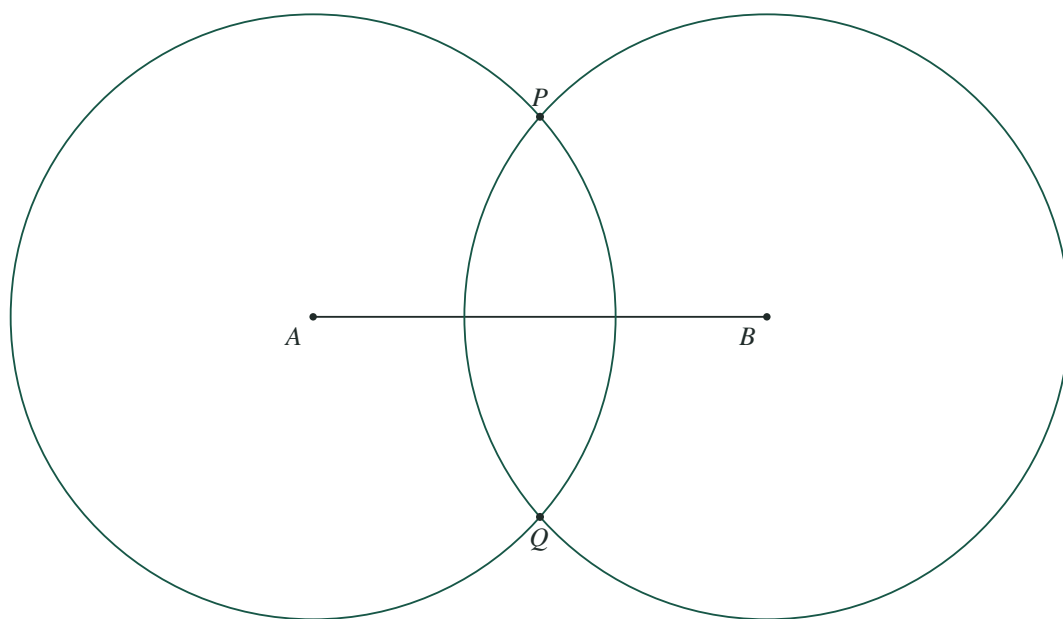
$$AP = 4 \text{ cm}$$

Use equal compass radii.

$$BP = 4 \text{ cm}$$

Two circle intersections

The centres are closer than the sum of the radii.



4 Combining conditions

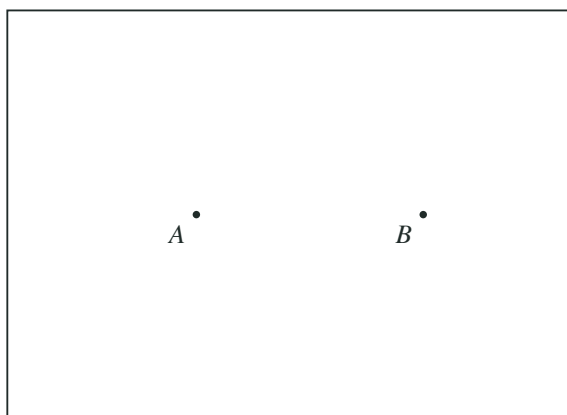
Q19 Construct all points 3 cm from both A and B . Mark them P and Q .

**Q20** $AB = 10$ cm. How many points are 3 cm from both A and B ? Explain.

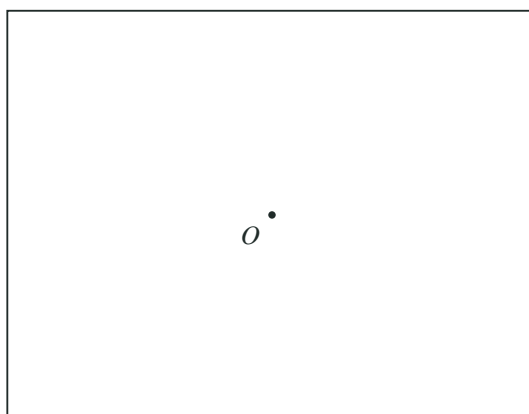
Q21 Construct all points 3 cm from both A and B . Label any points you find.



- Q22** Within the box shown, shade points less than 2 cm from A and nearer to A than B . Show excluded boundaries with dashed lines.



- Q23** Within the box shown, shade points at least 1 cm and at most 2 cm from O . Include both boundaries.



Q24 Construct all points equally far from A and B and exactly 5 cm from A . Mark them P and Q .



Solving principles

1. Translate each condition

Decide which circle, bisector or parallel line it describes.

2. Keep compass widths equal

Matching arcs establish equal distances.

3. Keep the construction visible

Show the arcs used to locate the line or point.

4. Test the chosen region

Check a point against every condition before shading.