

GCSE MATHS

Congruence and geometric proof

Worked Solutions

Q1

★

$$AB = DE$$

$$BC = EF$$

$$AC = DF$$

$$\triangle ABC \cong \triangle DEF (SSS)$$

All three corresponding sides are equal.

Apply the three-side test.

Q2

★★

$$AB = DE$$

$$AC = DF$$

$$\angle BAC = \angle EDF$$

SAS

Two side pairs are equal.

The equal angle lies between the given sides.

Use side-angle-side.

Q3

★★

ASA

Two matching angles and the corresponding side are equal.

Q4

★★

RHS

The right angle, hypotenuse and one other side match.

Q5

★★

AAA is not a congruence test

Matching angles fix shape but not size.

Q6

★★★

Not always

The third vertex may have two possible positions, giving triangles with different remaining sides and angles.

Q7

**

$$AB \leftrightarrow DE$$

A matches D and B matches E .

$$DE = 7 \text{ cm}$$

Corresponding sides are equal.

Q8

**

$$C \leftrightarrow F$$

Match the vertices in order.

$$\angle DFE = 52^\circ$$

Corresponding angles are equal.

Q9

**

$$A \leftrightarrow Q$$

Match the first and third vertices.

$$C \leftrightarrow R$$

$$AC \leftrightarrow QR$$

Join their matching endpoints.

Q10

$$3x + 1 = 16$$

Corresponding sides are equal.

$$x = 5$$

Subtract one, then divide by three.

Q11

$$2x + 3 = 5x - 9 \Rightarrow x = 4$$

Equate the corresponding sides.

$$2(4) + 3 = 11 \text{ cm}$$

Substitute back.

Q12

$$2x + 10 = 5x - 20 \Rightarrow x = 10$$

Corresponding angles are equal.

$$\angle A = 2(10) + 10 = 30^\circ$$

Calculate the requested angle.

Q13

★★★★

(a) $AB = AC$

$BM = CM$

$AM = AM$

$\triangle ABM \cong \triangle ACM \text{ (SSS)}$

Use the equal sides and the midpoint.

The side is common.

All three side pairs match.

(b) $AB = AC$

$BM = CM$

$AM = AM$

$\triangle ABM \cong \triangle ACM \text{ (SSS)}$

Three side pairs match.

Apply SSS.

$\angle BAM = \angle MAC$

Corresponding angles are equal, so AM bisects the angle.

(c) $\triangle ABM \cong \triangle ACM \text{ (SSS)}$

 $AB = AC$, $BM = CM$ and AM is common.

$\angle AMB = \angle AMC$

Corresponding angles are equal.

$2\angle AMB = 180^\circ$

The equal angles form a straight line.

$\angle AMB = 90^\circ$

Hence AM is perpendicular to BC .

Q14

★★★

$AB = AC$

Use the equal sides and shared side.

$AD = AD$

$\angle BAD = \angle DAC$

 AD is an angle bisector.

$\triangle ABD \cong \triangle ACD \text{ (SAS)}$

The angle is included.

$BD = DC$

Corresponding sides are equal.

Q15

★★★★

(a) $PA = PB$

$QA = QB$

$PQ = PQ$

$\triangle PAQ \cong \triangle PBQ$ (SSS)

Match the three side pairs.

Use SSS.

(b) $\triangle PAQ \cong \triangle PBQ$ (SSS)

Use $PA = PB$, $QA = QB$ and common PQ .

$\angle APQ = \angle QPB$

Corresponding angles at P are equal.

Q16

★★★

$\angle BAC = \angle DCA$

 AB is parallel to DC .

$\angle BCA = \angle DAC$

 BC is parallel to AD .

$AC = CA$

The diagonal is common.

$\triangle ABC \cong \triangle CDA$ (ASA)

Use two equal angles and their common side.

Q17

★★★

$\triangle ABC \cong \triangle CDA$ (ASA)

Alternate angles match and AC is common.

$AB = CD$

Corresponding sides of the congruent triangles are equal.

Q18

★★★★

$\angle BAO = \angle DCO$

 AB is parallel to CD .

$\angle ABO = \angle CDO$

Use the same parallel sides and the other diagonal.

$AB = CD$

Opposite sides are equal, as allowed.

$\triangle ABO \cong \triangle CDO$ (ASA)

The included corresponding sides match.

$AO = CO$

Corresponding sides are equal.

Q19

$$AO = CO$$

$$BO = DO$$

$$\angle AOB = \angle COD$$

$$\triangle AOB \cong \triangle COD \text{ (SAS)}$$

$$\angle ABO = \angle CDO$$

$$AB \parallel CD$$

Use the given bisected diagonals.

Vertically opposite angles are equal.

The equal angle is included.

Corresponding angles are equal.

Equal alternate angles imply parallel lines.

Q20

$$AB = AD$$

$$BC = CD$$

$$AC = AC$$

$$\triangle ABC \cong \triangle ADC \text{ (SSS)}$$

$$\angle BAC = \angle CAD$$

Use the kite sides and shared diagonal.

Three side pairs match.

Corresponding angles prove the bisection.

Q21

$$AB = AC$$

$$AE = AD$$

$$\angle BAE = \angle CAD$$

$$\triangle ABE \cong \triangle ACD \text{ (SAS)}$$

$$BE = CD$$

Match the side pairs.

Both are the angle BAC .

Use the included angle.

Corresponding third sides are equal.