

GCSE MATHS

Congruence and geometric proof

Long workbook

Questions on GCSE Maths topics, including challenges beyond typical exam questions.

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Topic recap

Congruent triangles

Congruent triangles have the same shape and size. Match vertices in the same order before comparing sides or angles.

Tests

SSS: three sides. SAS: two sides and their included angle. ASA: two angles and a corresponding side. RHS: a right angle, the hypotenuse and one other side.

Shared information

A side used by both triangles is equal to itself. A midpoint gives two equal lengths. Parallel lines give equal alternate or corresponding angles.

Proof

State the matching equal sides or angles, give the congruence test, then use corresponding parts to reach the required conclusion. Equal angles alone prove similarity, not congruence.

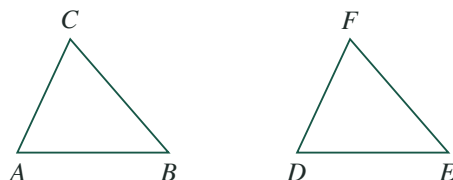
Use the stated lengths and relationships; do not measure the diagrams.

1. Congruence tests

Example Problem 1

**

$AB = DE$, $AC = DF$ and angle $BAC = \text{angle } EDF$. Explain why triangles ABC and DEF are congruent.



The idea

1. Check that the equal angle is between the two equal sides.

Worked Solution

$$AB = DE$$

Two matching sides are equal.

$$AC = DF$$

$$\angle BAC = \angle EDF$$

This is the included angle in each triangle.

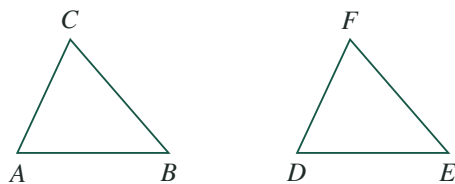
$$\triangle ABC \cong \triangle DEF \text{ (SAS)}$$

State the congruence test.

1 Congruence tests

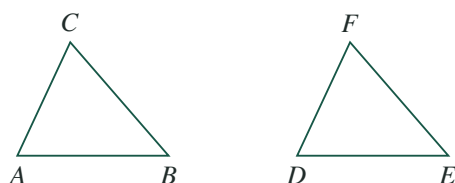
Q1 $AB = DE$, $BC = EF$ and $AC = DF$. Name the congruence test for triangles ABC and DEF .

*



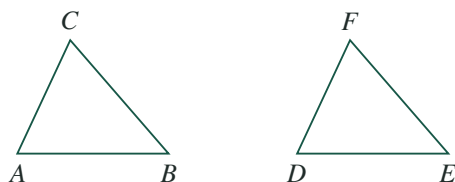
Q2 $AB = DE$, $AC = DF$ and angle $BAC = \text{angle } EDF$. Name the congruence test.

**



Q3 Angle $ABC = \text{angle } DEF$, angle $BAC = \text{angle } EDF$ and $AB = DE$. Name the congruence test.

**



Q4 Two right-angled triangles have equal hypotenuses and one equal shorter side. Name the congruence test. **

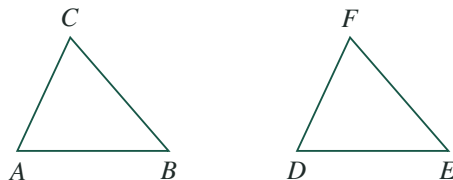
Q5 Two triangles have angles 40° , 60° and 80° . Must they be congruent? Explain. **

Q6 Two sides and a non-included angle match in two triangles. Is that always enough to prove congruence? Explain. ***

2. Corresponding parts

Example Problem 2

Triangles ABC and DEF are congruent in that order. $AB = 2x + 5$ cm and $DE = 17$ cm. Find x .



The idea

1. Equate corresponding sides before solving.

Worked Solution

$$2x + 5 = 17$$

$$x = 6$$

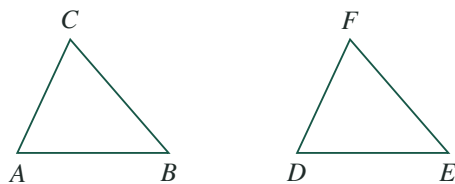
AB corresponds to DE .

Subtract five and divide by two.

2 Corresponding parts

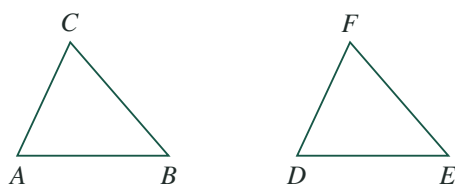
Q7 Triangles ABC and DEF are congruent in that order. $AB = 7$ cm. Find DE .

**



Q8 Triangles ABC and DEF are congruent in that order. Angle ACB is 52° . Find angle DFE .

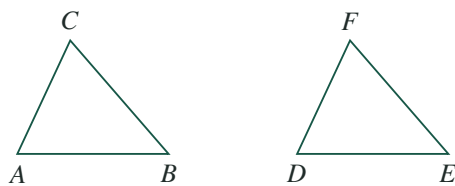
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Q9 Triangles ABC and QPR are congruent in that order. Name the side corresponding to AC .

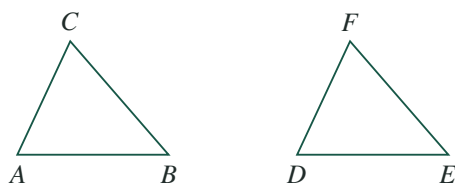
**

- Q10** Triangles ABC and DEF are congruent in that order. $BC = 3x + 1$ cm and $EF = 16$ cm. Find x . ***



- Q11** Two congruent triangles have corresponding sides $2x + 3$ cm and $5x - 9$ cm. Find their common length. ***

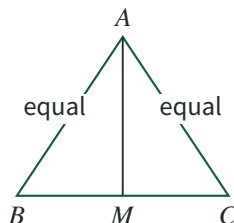
- Q12** Triangles ABC and DEF are congruent in that order. Angle A is $(2x + 10)^\circ$ and angle D is $(5x - 20)^\circ$. Find angle A . ***



3. Isosceles triangles and symmetry

Example Problem 3

$AB = AC$ and M is the midpoint of BC . Prove angle $ABM =$ angle ACM .



The idea

1. Use the shared side to complete an SSS argument.

Worked Solution

$$AB = AC$$

Three side pairs match.

$$BM = CM$$

$$AM = AM$$

$$\triangle ABM \cong \triangle ACM \text{ (SSS)}$$

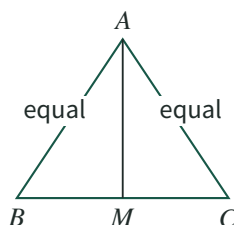
Apply SSS.

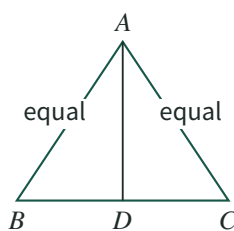
$$\angle ABM = \angle ACM$$

Corresponding base angles are equal.

3 Isosceles triangles and symmetry

Q13 $AB = AC$ and M is the midpoint of BC .

(a) Prove triangles ABM and ACM are congruent.**(b)** Prove AM bisects angle BAC .**(c)** Prove AM is perpendicular to BC .**Q14** $AB = AC$. Point D lies on BC and AD bisects angle BAC . Prove $BD = DC$.

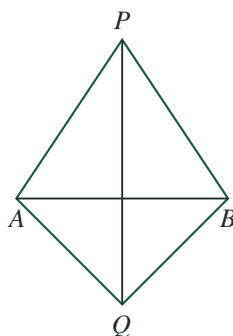


Q15 P and Q lie on opposite sides of AB . $PA = PB$ and $QA = QB$.

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(a) Prove triangles PAQ and PBQ are congruent.

(b) Prove PQ bisects angle APB .

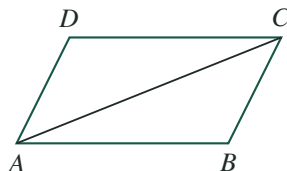


4. Quadrilaterals and longer proofs

Example Problem 4

★★★★

$ABCD$ is a parallelogram. Prove $AD = BC$ using diagonal AC .



The idea

1. Find two pairs of alternate angles and use the shared diagonal.

Worked Solution

$$\angle BAC = \angle DCA$$

$$\angle BCA = \angle DAC$$

$$AC = CA$$

$$\triangle ABC \cong \triangle CDA \text{ (ASA)}$$

$$BC = DA$$

AB is parallel to DC .

BC is parallel to AD .

The side is common.

Match the triangles in this order.

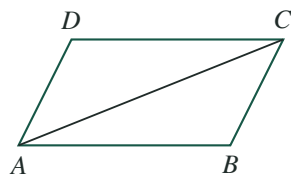
Use corresponding sides.

4 Quadrilaterals and longer proofs

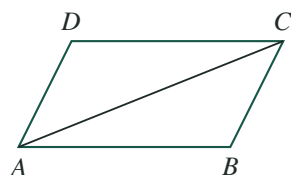
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Q16 $ABCD$ is a parallelogram. Prove triangles ABC and CDA are congruent.

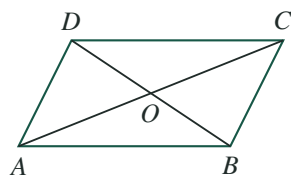
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**Q17** $ABCD$ is a parallelogram. Use congruent triangles to prove $AB = CD$.

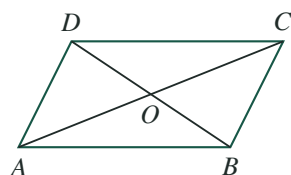
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Q18 The diagonals of parallelogram $ABCD$ meet at O . Prove $AO = CO$. You may use $AB = CD$.

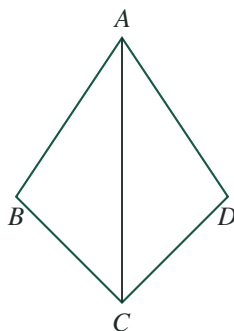


Q19 In quadrilateral $ABCD$, diagonals AC and BD meet at O . $AO = CO$ and $BO = DO$. Prove AB is parallel to CD .



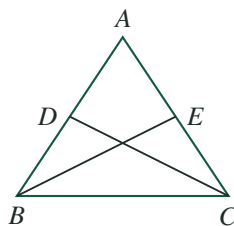
Q20 $ABCD$ is a kite with $AB = AD$ and $BC = CD$. Prove AC bisects angle BAD .

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Q21 $AB = AC$. D lies on AB and E lies on AC , with $AD = AE$. Prove $BE = CD$.

★★★★



Solving principles

1. Choose the triangles

Write the vertices in matching order.

2. Use the given information

Distinguish stated facts from what you are trying to prove.

3. Check the included angle

For SAS, the angle must lie between the two given sides.

4. Finish the argument

Name the corresponding sides or angles that establish the requested result.